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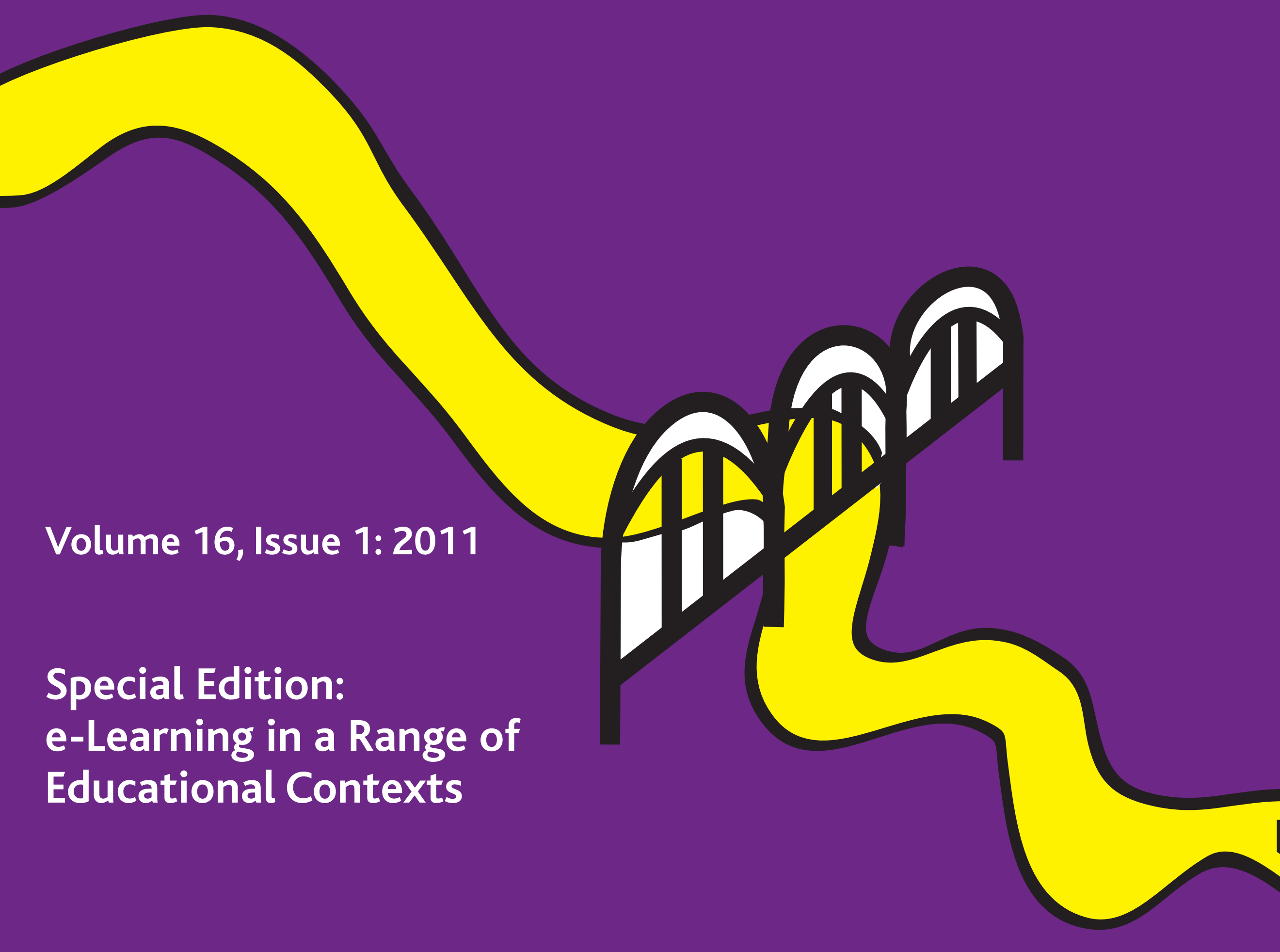

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Correspondence and articles for review should be addressed to: Research Manager, Wilf Malcolm Institute of Educational Research, Faculty of Education, The University of Waikato, Private Bag 3105, Hamilton, 3240, New Zealand. Email: wmier@waikato.ac.nz

Business correspondence: Orders, subscription payments and other enquiries should be sent to the Administrator, *Waikato Journal of Education,* Wilf Malcolm Institute of Educational Research, Faculty of Education, The University of Waikato, Private Bag 3105, Hamilton, 3240, New Zealand, Email: wmier@waikato.ac.nz

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Processing mathematics through digital technologies: A reorganisation of student thinking?

Dr Nigel Calder
Faculty of Education
University of Waikato

Abstract

This article reports on aspects of an ongoing study examining the use of digital media in mathematics education. In particular, it is concerned with how understanding evolves when mathematical tasks are engaged through digital pedagogical media in primary school settings. While there has been a growing body of research into software and other digital media that enhances geometric, algebraic, and statistical thinking in secondary schools, research of these aspects in primary school mathematics is still limited, and emerging intermittently. The affordances of digital technology that allow dynamic, visual interaction with mathematical tasks, the rapid manipulation of large amounts of data, and instant feedback to input, have already been identified as ways mathematical ideas can be engaged in alternative ways. How might these, and other opportunities digital media afford, transform the learning experience and the ways mathematical ideas are understood? Using an interpretive methodology, the researcher examined how mathematical thinking can be seen as a function of the pedagogical media through which the mathematics is encountered. The article gives an account of how working in a spreadsheet environment framed learners' patterns of social interaction, and how this interaction, in conjunction with other influences, mediated the understanding of mathematical ideas, through framing the students' learning pathways and facilitating risk taking.

Keywords

Mathematical thinking, digital technologies, affordances, spreadsheets

Introduction

Information and Communication Technology (ICT) offers potential for transforming the nature of the learning process. The learning environment and the manner in which learners engage in tasks differ, with consequential variation in both learner activity and dialogue compared to other pedagogical media. The Internet, for instance, offers greater

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scope for child-centred, inquiry-based learning. It has enabled learners to connect with an extensive, eclectic array of information, opinion and expertise, albeit varying in quality. This variation itself has changed the emphasis of particular aspects of learning. Navigating these information pathways emphasises a different set of skills and ways of thinking, giving privilege to alternative approaches to learning. The need to evaluate, differentiate and synthesise becomes critical for the learner to discern the appropriateness of information.

Meanwhile, when mathematical tasks are encountered through ICT media the learner frames the interaction with the task from a distinct perspective. A digital pedagogical media might enhance or constrain the learning experience in particular ways. The affordances offered, for instance, through the linking of symbolic, tabular and visual representations of the same phenomena, the virtually instantaneous response to the input of data, and the potential for visual reasoning (Borba & Villarreal, 2005; Smart, 1995; Tall, 2000) have all been identified and examined in various contexts, through a range of digital pedagogical media. These give rise to more generic entitlements: learning from feedback; observing patterns; seeing connections; working with dynamic images; exploring data; and "teaching" the computer, which have also been recognised as opportunities students can expect through engaging school mathematics through ICT media (Johnston-Wilder & Pimm, 2005).

Research into the way dynamic geometry software, such as *Cabri-geometry*, shapes students' understanding of geometric concepts (e.g., Laborde, 1995); the influence of CAS on learning in algebra (e.g., Kieren & Drijvers, 2006); the suitability of spreadsheets for visualisation of number patterns (e.g., Calder, 2004), and an interactive approach (e.g., Beare, 1993) has been undertaken. Yet, there is a scarcity of research on how, as a media for exploration, the spreadsheet might influence the dialogue, the investigative pathway, and hence the understanding, of students. This was a key focus of the study. Digital technologies offer new perspectives on the engagement of learners and the ways they might actually negotiate their understanding. The focus was on how understanding might be shaped when spreadsheets were used as a tool for exploring mathematical problems.

The place of discourse and the way understanding evolves through the differing media is a key aspect of this discussion. This also involves theoretical perspectives such as hermeneutics (e.g., Ricoeur, 1981), its relationship with education (e.g., Gallagher, 1992), and with mathematics education (e.g., Brown, 2001), allied with pedagogical perspectives that have evolved from interaction in the ICT environment per se. The processes involved with learning, including an examination of conceptualisation and what a "concept" might be, and how it emerges, are also critical to this undertaking. Learning mathematics, as Brown (2001) contends, is "a perpetual state of becoming, governed through the social discourses, enacted through the individual" (p. 173). In this ascribed interpretation of learning, "concepts" are not fixed realities from which we peel the outer layer to reveal their entirety, but are more elusive, formative processes that become further enriched as learners use their temporary fixes to view events from fresh, ever-evolving perspectives. The objectification of knowledge is a progressive process of noticing; an active, creative and interpretive social process that surfaces through the interaction of a range of elements such as language, symbols and artefacts (Radford, Bardini, & Sabena, 2007). In essence, the mathematical task, the pedagogical medium, the preconceptions of the learners and the dialogue evoked are inextricably linked. It is from their relationship with the learner that understanding emerges. This understanding is the learner's interpretation of the situation through those various filters.

As the ICT metamorphosis is rapid, it is challenging for researchers to evaluate the effects on learning, in contemporary settings. The time lag between the dissemination of research findings, coupled with the synthesis of various studies required to build a meaningful picture of the influence of ICT in the learning process for mathematics education, and the rapidly changing, commercially driven nature of software and hardware development, can lead to the technology being superseded before a coherent analysis of its implications has emerged. Yet it is critical that this research is undertaken, so a pattern of implications can evolve, and be recorded. It is also true that generalisations may emerge. This study was part of an extended examination of the ways using spreadsheets as the pedagogical medium for investigating mathematics might restructure the learners' understanding of mathematical ideas. It reports on a study that examined 10-year-old students' use of spreadsheets to investigate mathematical problems. It is, however, situated in the broader frame of using digital pedagogical media in mathematics education generally. While there are diverse interpretations of e-learning that encompass both particular processes and pedagogical media, for the purposes of this article we are situating our perspective in the processing of mathematical phenomena through the digital pedagogical medium of the spreadsheet.

Research has identified ways that using a spreadsheet to investigate mathematical tasks has influenced the generation of sub-goals. For example, their potential to open up investigative opportunities (Beare, 1993; Calder, 2004; Drier, 2000), to enhance students' ability to model mathematically (Zbiek, 1998), and to enrich students' ability to problem solve and communicate mathematically (Chance, Garfield, & delMas, 2000). Providing an environment to test ideas, link the symbolic to the visual, link the general to the specific, give almost instantaneous feedback to changing data, be interactive, and give students a measure of autonomy in their investigation are other opportunities afforded that facilitate an investigative approach. The current study was designed to explore how the pedagogical medium of a spreadsheet, used as a tool for investigation, might have influenced the learning experience and how processing mathematics in this way might have reorganised children's mathematical perceptions and understandings.

Methodology

Hermeneutics, the study of interpretation, offers a way to better understand a localised learning situation. Central to this are the worlds of both the participants and the researcher. Just as the participants bring their historically situated preconceptions and discourses to each situation, so too does the researcher. There are multiple versions and interpretations of situations, and multiple perspectives from which these interpretations are evoked. As such, the space the researcher occupies in each version of their interpretation is as much a part of the data as the observations themselves (Brown, 2001; Mason, 2002). Implicit to this interpretive approach are limitations associated with the researcher's perspective—any interpretation will be the researcher's version of events and as such they are socially and historically situated as the data itself is. Narrative frameworks, from which accounts are fabricated, temporarily fix these historically positioned interpretations, allowing the provisional interpretations at the local level. The students' understanding evolved through iterations of engaging with the tasks and the consequential repositioning of their perspectives. These modified perspectives in turn framed the subsequent re-engagement with the tasks. In a version of mathematical learning flavoured by this perspective, mathematics is a social construct premised on previous interpretative stances (Brown, 2001). From this perspective, an examination of the learners' preconceptions, their interpretations (as manifest in their dialogue and actions) and how they subsequently re-engaged with the activities gave insights into the layering of meaning as their understanding evolved.

One purpose of this study was to identify the ways participants approached the mathematical investigations as they negotiated the requirements of the tasks, and how this might have filtered their conjectures and generalisations. Central to this is the participants' dialogue as they negotiated the meaning of the tasks. By examining the participants' verbal interactions as they engaged in the tasks, by observing their actions, and by analysing their reflections, insights were gained into the ways investigating mathematical problems with a spreadsheet might have influenced their understanding of the problem. As they negotiated the requirements of the tasks and explored possible solutions, a more fulsome picture of the ways participants framed their conjectures and generalisations emerged.

Participants and research methods

The participants for this particular analysis were drawn from year 6 students attending five partnership schools associated with the University of Waikato at Tauranga campus. They were at the time involved in a collaborative project offering programmes to develop gifted and talented students in their schools. There were four students from each school (five from one school), who had been identified through a combination of problem-solving assessments and teacher reference. The 12 boys and nine girls came from a range of socio-economic backgrounds. Two of the schools were full primary (years 1–8) schools, while the other three included students from years 1 to 6. The participants were located in a classroom situation that included seven computers with spreadsheets as available software. This was the typical working environment for two of the schools, while the other three schools had three or four computers in each class at this level. The group was familiar with this particular classroom, having worked there on previous occasions that year, engaging in rich mathematical tasks and investigations as part of their programme.

For the research project, the students worked on a programme of activities using spreadsheets to investigate mathematical problems, predominantly suitable for developing algebraic thinking. They were observed, their conversations were recorded and transcribed, and their investigations were printed out or recorded. There were school group interviews, and interviews with working pairs. They undertook a survey based on opinion and motivational considerations, while some ongoing data was also gathered over a longer-term period (18 months) with three of the groupings.

The emergence of sub-goals and mathematical thinking

The ways students made initial sense of an investigative situation when approaching it through the pedagogical medium of the spreadsheet, and how subsequent learning trajectories are conditioned by those initial exchanges were considered in this section. It examines the approaches in which participants engaged, and how their preliminary responses were shaped, and their sub-goals framed, by the features of the spreadsheet setting. It also explores the manner in which this might have filtered their understanding and conjectures.

Investigation of a mathematical situation, whether one contrived as a "school maths" model or one necessitated by real life circumstances, requires an aspect of familiarisation. Polya (1945) was the first to formally articulate this "understand the problem" stage in his four-step approach to problem solving, but contemporary mathematics educators maintain the validity of this initial step (Holton, 1998). What am I trying to find out? What information do I have? How do I gather more pertinent information? What picture is beginning to emerge? These questions may be part of that familiarisation process, and the individual's response to the mathematical phenomena that will condition the shape of the investigative process.

This familiarisation process isn't distinct from the solving process, however, nor is it necessarily chronologically placed prior to the commencement of that process. Nunokawa (2001), discussing Resnick's concept of sub-goals in solving more complicated problems, observed that these aspects were intertwined. He noted that the settlement of sub-goals was conditioned by the learner's understanding of the situation, but also that the sub-goals settled on by the learner influence her interpretation of the problem situation. Sub-goals are generated as part of the familiarisation and re-familiarisation of the problem, and where the learning is situated will influence the specificity of their production. The data in this section illustrated a cyclical process as the students interpreted the task from their preconceptions, borne of their prevailing discourses, engaged with the task, evoking a subsequent reorganisation of their thinking, and then reinterpreted from these fresh perspectives.

If engaging the mathematical tasks through the spreadsheet medium permitted the learner alternative ways of envisioning the intentions of the task and then navigating the investigative process in particular ways, it is reasonable to assume that their thinking was conditioned by these alternative engagements. The actions of the students and the accompanying dialogue were examined in an extended excerpt to help determine how the learning trajectory might have evolved as the students' gaze moved between their underlying perceptions and interaction with the task. Consideration was given to whether the sub-goals they articulated through their interactions were shaped in particular ways by the pedagogical medium.

The first set of data refers to an activity based around exploring the number patterns formed when multiplying numbers by 101, the 101 times table activity (see Table 1). The data exemplifies the typical investigative approach undertaken by the students. The student dialogue is interspersed with associated analysis and responses from the interviews.

Table 1. 101 times table task

101 times table
Investigate the pattern formed by the 101 times table by
• Predicting what the answer will be when you multiply numbers by 101
• What if you try some 2 and 3 digit numbers? Are you still able to predict?
• Make some rules that help you predict when you have a 1, 2, or 3-digit number. Do they work?
• What if we used decimals?

It was noticeable that the pupils were willing to immediately enter something into the spreadsheet. There was little attempt, in general, to negotiate the task situation through discussion or pencil and paper methods, although some individual processing of the task requirements must have occurred. For example:

Awhi: So we've got to type in 101 times.

Ben: How do you do times?

Awhi: There is no times button. Oh no, wait, wait, wait.

Ben: There is no times thing. Isn't star?

Awhi: =A1*101. Enter.

This approach of immediately entering input into the spreadsheet was confirmed with responses in the interviews such as the ones below.

Awhi: I preferred thinking something about what I needed to do, then take it and highlight it down and then the whole table is there, which would help me.

Adam: What we did is we tried a few formulas. To start off with we like typed in a few formulas that we thought it might be, and then went through and got the correct one, which got us the right answers.

It appeared the actual spreadsheet environment provided the impetus to take this initial approach. Another pupil commented:

Dee: Because of the spreadsheet, we went straight to formulas, looked for a pattern, for a way to make the spreadsheet work.

The use of spreadsheets led them to an immediate form of generalisation. To generate a formula that models a situation is to generalise in its own right, but to consciously look to *Fill Down* ("highlight it down"), or create a table of values was also indicative of an implicit cognisance of a pattern; of an iterative structure that was a way into exploring the problem.

Awhi and Ben continue:

Ben: 202.

Awhi: Now let's try this again with three. OK, what number do you think that will equal? 302?

Ben: No, 3003.

They drew on their prevailing discourses in several interrelated areas: number structure and patterns, number operations, and the spreadsheet environment. As they attended to their activity associated with the task, their understandings and persuasions from these individual broader frames had influenced which aspects were given primacy in the process of predicting. They copied the formula down using the *Fill Down* function to produce the output below (Table 2).

Table 2. Output from Fill Down

101
202
303
404
505 etc.

Ben: Oh no, 303.

The output was different from the predictions that their prevailing discourse had framed. The pupils appeared to use the table structure as a means to interpret the situation. It allowed them to more easily notice the relationship between the input and the output, and the ensuing pattern of the output values. Their perspective evolved and they re-engaged with the task from a fresh, modified stance.

Awhi: If you go by 3, it goes 3 times 100, and zero, and 3 times 1; 303.

The pattern that Awhi articulated was consistent with the output that the spreadsheet produced. Their informal proposal was confirmed and they reset the direction of the investigative trajectory accordingly. They were immediately into the business of predicting and confirming in a confident, relatively uninhibited manner. They explored a range of two-digit numbers. They began to pose conjectures, and test them in an informal approach:

Awhi: OK. Now you try a number.

Ben: My lucky number 19.

Awhi: That'll be one thousand, nine hundred, and nineteen.

Ben: Equals. So we need to think of a rule.

Awhi: It's like double the number. It's 19, 19.

Ben: What about 20? Oh you'll get 2020.

They appear to have predicted what the product would be when nineteen was multiplied by one hundred and one by utilising the patterns that were beginning to emerge for them. They confirmed their prediction ("Equals") before attending to a more generalised account of the situation. Ben then used their emerging informal conjecture ("double the number") to pose and confirm a further prediction. The ability to predict, form a conjecture then test it is indicative of a robust generalisation process. In this case, and with others in the study, the children chose a particular path because they were using the spreadsheet. The shape of their investigation was determined by the particular pedagogical approach. They were also able to quickly move beyond the constraints of the prescribed task, forming a fresh generalisation. They had reset the sub-goal of the investigation and were exploring the effect on a four-digit number.

Awhi: Oh try 1919.

Ben: I just have to move that little number there, 1919.

The following output is produced.

193819

Interestingly, they seemed to disregard this output and form a prediction based on their previous interpretations. Their interpretation, underpinned by their prevailing discourses in the associated domains, superseded the output, or influenced their noticing: what they brought to the foreground.

Awhi: Now make that 1818, and see if its 1818.

Ben: Oh look 18, 3, 6, 18.

There was an unexpected output, which made them re-engage in the activity, reflect on the output and attempt to reconcile it with their current perspective. It caused them to reshape their emerging conjecture.

Awhi: Before it was 193619: write that number down somewhere (183618) and then we'll try 1919 again.

Ben: Yeah, see 19, 3, 8, 19. Oh, that's an 8.

Awhi: What's the pattern for two digits? It puts the number down first then doubles the number. This is four digits. It puts the number down first then doubles, and then repeats the number.

The data indicated that the pupils engaged a local hermeneutic process as they familiarised themselves with the task then moved between their broader perspectives and engagement with the task. They interpreted the task from their preconceptions in the associated domains, then influenced by the affordances of the pedagogical medium, engaged with the task. This engagement shifted their perspective in varying degrees: their viewpoint was modified; they set fresh sub-goals in the investigative process, and re-interpreted the task from these fresh perspectives. Each re-engagement transformed their underlying discourse to some extent. In this way their understanding was an ongoing process that emerged from evolving interpretations through this iterative process.

The data also suggested they were using a visual referent for the theory that was evolving. They were looking at the actual visual sequence itself that was producing the number patterns. The naming of the products as a visual sequence, for example, 18, 3, 6, 18 by Ben in the above transcript supported that interpretation. They were seeing the number as three or four discrete visual elements, rather than thinking of a consequence of an operation. Meanwhile, once more the data indicated that the spreadsheet environment influenced their approach to the investigation. It filtered the path to, and the nature of, their conjectures, with their subsequent interpretations shaped in visual rather than procedural terms. Their understanding emerged from these interpretations as they engaged with the task through their various underlying perspectives. The setting of the sub-goals was influenced by the visual tabular structure of the spreadsheet output and appeared to organise their thinking so that their generalisations and understandings were shaped in a manner that was specific to this environment. The students' learning trajectory evolved differently through the spreadsheet medium, facilitating a reorganisation of their thinking and as a consequence their interpretations and understanding.

Risk Taking

Central to the processes of enquiry and investigation is the willingness to test ideas and to experiment and explore in confident, relatively unrestrained manner. The students' willingness to take risks and try things when engaging the mathematical activities through the medium of the spreadsheet was evident in the data. Also linked to this, was the interest and enjoyment gained by the challenging aspects evoked by the pedagogical medium. This was at times linked to the students having more space for the reflective process, but at others to the students viewing the data from alternative perspectives. This second point resonates with the discussions regarding the initial engagement and the tabular structure. Student confidence appeared to be enhanced by their enjoyment and an interested disposition, while confidence enhanced the learner's propensity to take risks.

By implication, problem solving contains an element of the unknown that requires unravelling and addressing through the application of strategies in new situations or in an unfamiliar manner. This requires a degree of creativity and a willingness to take conceptual or procedural risks of a mathematical nature. It is risk taking in a positive, creative sense as compared to risky behaviour. The data appeared to indicate a greater propensity for exploration and risk taking engendered by the spreadsheet environment. This is consistent with other findings (Sandholtz, Ringstaff, & Dwyer, 1997; Calder, 2006). The responses seem to be primarily related to the functional or formatting affordances of the spreadsheet. For example:

Fran: Using a spreadsheet made it more likely to have a go at something new because it does many things for you. You have unlimited room. You can delete, wipe stuff out.

Tony: It was easy to try things—saved you rubbing it out, you press delete and it's gone. What else was good about it? Trying things out.

Ben: We tried a couple of formulas and none of them were right but we could see what the formula might be, so we could change it around a bit.

Ant: Yeah, like when we had to on the first activities when Dan had 8 then he had 11 we had to find what was different—we could try things out and see if that worked and change it.

Sophie: I always find it good for me. I can put something in and if it's not quite right, I can change a couple of things and bang, it changes it automatically and I don't have to start from the beginning again.

These student comments reflect a certain comfort with trying things, knowing they can be easily modified, and with an awareness of the rapidity of that modification process. It seemed there was an implicit reference to the encouragement of experimentation as well, through the facility to model situations in various ways, for example formulae or tables, coupled with that ease and speed of modification of those models. They also appeared to be more able to easily experiment with new ideas that arose during the process, as illustrated by the comments below.

Whitu: Through doing the work I found about the power of ten and tested it out; used it.

Deanna: Good for ones like tracking the money, but if you forget something in the formulas that's wrong, you can just change it; and you can enter future ones to see what will happen.

Some used the spreadsheet for more usual investigative approaches but nevertheless found the spreadsheet conducive to that practice. For instance:

Bree: It took a while to figure out what we were doing. How did we solve the problem? It was trial and error. We did one digit, two digits, three digits. It was in the three digits that we started to figure out what the pattern was. There was a lot of trial and error. What if we try this number? What if we try that number?

Tony: Me, and a guy Thomas, we were playing around with the graphs and you could find out what different graphs are used for.

The following comments were indicative of responses illustrating a generic benefit of the spreadsheet as an investigative tool and indicated the propensity that is engendered by the medium for a confident investigational approach and a willingness to take risks.

Beth: I looked at how it was written down and look at all the patterns; then I sorted it out in my head then put it down, and if it wasn't right then try another one. Experiment.

Ella: I found it helpful that it could calculate itself, and I had more time to work on the problem.

Aspects related to investigating in the spreadsheet environment, such as the tabular format for output, the immediacy of the response to input, the facility to compute large amounts of data simultaneously, and to modify various elements quickly and easily, all engendered confidence in students to try things and take risks. Confidence is a very personal condition, however, and is borne of a layering of interactions and interpretations, some seemingly unrelated to the situation in which the researcher might have noticed the confidence or lack of confidence. Two people given the identical spreadsheet experience would have distinctive responses invoked by the experience. One student might feel very confident to try new approaches, and another not at all confident. Nevertheless, the environment had the potential to enhance the students' willingness to take risks. It was also a relatively non-threatening, easily managed environment. This would also seem to make it suitable for encouraging risk taking.

Conclusions

The distinctive nature of engaging mathematical tasks through the pedagogical medium of the spreadsheet framed the ongoing interactions, interpretations and explanations as the students envisioned their investigation through that particular lens. The actual learning trajectories were shaped by that initial engagement of creating formulas or columns and tables of data to model the mathematical situation. Digital technologies are generally more conducive to the modelling of mathematical situations than pencil-and-paper media, and the data were illustrative of the spreadsheet enhancing this aspect. The capacity to manipulate large amounts of data quickly, coupled with the potential for symbolic, numerical and visual representations enabled the students to produce models that could be observed simultaneously, with the links and relationships between them explored in an interactive manner. The students' interaction with alternative representations promoted learning through the comparison or combination of representations, enabling broader perceptions than what might have been gained from a single representation. As well, when the students were required to relate different representations to each other, they had to engage in activity such as dialogue, interpretation, and explanation that enhanced understanding.

The spreadsheet environment was also influential in the generation of sub-goals as the students' learning trajectories unfolded. As they alternated between attending to the activities from the perspective of their underlying perceptions, and then reflecting on this engagement with consequential modification of their evolving perspectives, they set sub-goals that plotted their ongoing interaction. These were frequently reset in response to the output generated within the spreadsheet environment. Sub-goals were generated at times because of opportunities afforded by the particular pedagogical medium. As well as those attributes that facilitated the modelling process, the facility to test immediately and reflect on emerging informal conjectures gave potential for the sub-goals the students set being shaped by the medium. The data indicated this. The data also demonstrated how the students' interpretations of the situations they encountered were influenced by the visual, tabular structure. It allowed more direct comparison of adjacent columns and enabled them more easily to perceive relationships between numerical values on which to base their new sub-goal, often linked to an emerging informal conjecture. It enhanced their ability to perceive relationships and recognise patterns in the data. Seeing the pattern evoked questions. On occasion the students pondered why the pattern was there, and what was underpinning a particular visual sequence.

While investigating in this environment, the students learnt to pose questions and sub-goals but also were encouraged to create personal explanations, explanations that were often visually referenced probably due to the pedagogical medium. It also gave opportunity through its various affordances for the students to explore powerful ideas and to explore concepts that they might not otherwise be exposed to. At times the learning trajectory evolved in unexpected ways. When the output varied, sometimes markedly, from what was expected, it caused tension that often led to the resetting of the sub-goal and substantial shifts in the way the student interpreted or engaged the situation. This aspect and other affordances including the interactive nature of the environment also appeared to stimulate discussion. The students wanted to verbally articulate the rapidly generated output and discuss the connections they could see, not least when it was unexpected.

Engaging the mathematical phenomena through a pedagogical medium that allowed the students to test informal conjectures, while being interactive and giving immediate feedback, enhanced the students' willingness and propensity to employ an investigative approach. They appeared to be more willing to take risks. The learners were willing to try fresh strategies in their approach to investigation and problem solving, The data were indicative of the spreadsheet environment affording learning behaviours and responses that facilitated the learner's willingness to take risks while operating within an investigative cycle. This seemed to allow the students to pose informal conjectures, to explore then reflect on them before, perhaps after several investigative iterations, either validating or rejecting them. The offering and investigation of informal conjectures fostered mathematical thinking.

When the students observed a pattern or graph rapidly, they were comfortable exploring variations in the patterns and, perhaps with teacher intervention, learned to make conjectures, and then pose questions themselves. This facility to immediately test predictions, reflect on outcomes, then make further conjectures, not only enhanced the students' ability to solve problems and communicate mathematically, but developed their logic and reasoning as the students investigated variations, or the application of procedures. Others have reported improved high-level reasoning and problem solving linked with this capability (Chance et al., 2000; Sandholtz et al., 1997). As well, the students using the spreadsheets progressed more quickly into exploring larger numbers and decimals. This appeared to indicate a greater propensity for exploration and risk taking engendered by the spreadsheet environment. Yet, although that is consistent with other findings (Calder, 2005; Sandholtz et al., 1997), only tentative links between using spreadsheets and greater mathematical risk taking can be drawn from this study.

While the data were illustrative of the spreadsheet, acting as a pedagogical medium, offering a distinctive environment for the students to investigate mathematical phenomena and influencing their actual learning trajectories, they also gestured towards it having transformative qualities; of it behaving as a conduit for the reshaping or reorganisation of their thinking.

The reorganisation of mathematical thinking and understanding

An aspect the data highlighted regarding the reorganisation of thinking was the nature of the students' initial engagement. Their approach was distinctive from the students in a classroom situation in that they immediately explored symbolic and tabular models of the situation—frequently with multiple, structured output, rather than a single numerical example. This framed the subsequent investigation of the mathematical activities, flavouring the investigative process, the generalisation of sub-goals, and the explanations with this distinguishing perspective. Their dialogue also contained phrases and meanings particular to the medium. Investigating by processes such as *Fill Down* or using a spreadsheet formula offered an alternative exploratory landscape with potential for the understanding to emerge in restructured ways. The speed and varying representations of feedback were also influential in the rearrangement of the students' methods and restructuring of the manner in which their learning trajectories and understandings evolved.

The spreadsheet environment reshaped the students' approaches, and the manner in which they traversed their actual learning trajectories, by the particular nature of their experiences while working within that environment. It allowed them to engage in alternative processes and to envisage their interpretations and explanations from fresh perspectives. The mathematising facilitated by the medium was transformed by the visual, interactive nature of this investigative process. There was a visual perspective to their mathematical thinking, while the visual tabular structure enhanced the possibility of seeing relationships in ways that might otherwise have been unattainable or inaccessible. Coupled with other affordances, such as the increased speed of the feedback, this gave them confidence to take risks and expanded the boundaries of what constituted mathematical knowledge, giving students access to ideas earlier than teachers' usual expectation. It allowed a shift in focus from calculation techniques to a focus on mathematical thinking and understanding. The distinctive influence of the spreadsheet environment on the investigative process as the students modelled the situations with visual representations, and the fostering of risk taking the environment engendered, appeared to enhance the students' facility to think mathematically and generalise. Subsequently, a reorganisation of the learners' thinking was facilitated.

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