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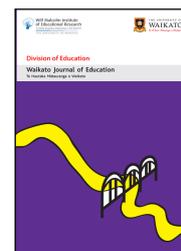
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Young children are natural inquirers: Posing and solving mathematical problems

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Abstract

Carefully observing young children at play in a mathematically rich environment has led me to reflect on the way children naturally pose and solve interesting mathematical challenges. Here, three examples of the playful learning of six-year-old children illustrate the problem solving and persistence children can display. Teachers are encouraged to foster children's problem posing by providing opportunities for children to engage with playful mathematics, planning time for children to pose and to solve their own problems, and watching and listening but intervening only to inspire children's mathematical investigations.

Keywords

Inquiry; mathematics; problem solving, playful learning in mathematics.

Abstract

Over the course of several research projects and through the generosity of teacher-colleagues, I have had the opportunity to visit classrooms to observe many children learning mathematics. Recently, in a German kindergarten, I was impressed by the determination of a six-year-old to solve several number challenges that she set for herself. Her concentration and persistence were notable. I could not get her out of my mind, and her behaviours made me think of other children I have met who were just like her in their application to a problem they had posed for themselves. I began to ask myself the question, what do I know about the natural inquiry and mathematical problem-solving propensity of young children?

In this paper I will describe several events that I have observed with children of similar ages. These events were selected for their explanatory power and are not meant to be exemplary or to reflect ideal classroom practice. From these examples, I will draw out some general features of the contexts and reflect on factors that foster mathematical curiosity and problem solving in young children.



Background

Cognitive developmental and educational research have shown that young children develop a relatively powerful ‘everyday’ or ‘informal’ mathematics that has several important characteristics (Clements & Sarama, 2007; Ginsburg, Cannon, Eisenband, & Pappas, 2006; Kibbe & Feigenson, 2015). One such characteristic is that young children have a spontaneous and sometimes explicit interest in mathematical ideas (Ginsburg & Amit, 2008; Rathé, Torbeyns, Hannula-Sormunen, Smedt, & Verschaffel, 2016). Naturalistic observation has shown that young children in their ordinary environments enjoy counting up to, what are for them are, large numbers (Irwin & Burgham, 1992), and are interested to know what is the ‘largest number’ and count on their fingers (Gelman, 2000; Soylu, Lester, & Newman, 2018). Mathematical ideas pervade children’s play (Lee, 2012b; Worthington & van Oers, 2016). Often young children spend time determining which block tower is taller, creating and extending block patterns, exploring shapes, and creating symmetries (Seo & Ginsburg, 2004). Much of this activity is spontaneous and occurs without adult guidance (Helenius et al., 2014). Often adults are unaware that children are playing in these ways (Fosse, , 2018; Svensson, 2015). As stated by Gelman (2000) “e can think of young children as self-monitoring learning machines who are inclined to learn on the fly, even when they are not in school and regardless of whether they are with adults” (p. 26).

Vygotsky (1978) argued that the play context is essential for development. He suggested that because play is child-initiated, it allows children to have control of their own learning. Studying children’s play as particular events can empirically ground theoretical constructs; in addition, theoretical constructs influence the interpretation of particular events (Erickson, 2012). I hold a theoretical perspective of learning informed by those of Vygotsky and subsequently Ernest, who recognised that knowing is an active process that is, “individual and personal, and that it is based on previously constructed knowledge” (Ernest, 1994, p. 2). A socio-constructivist theory of learning (von Glasersfeld, 1987) also holds that knowledge is not fixed; rather it is socially negotiated and is sought and expressed through language. It is a theory in which “learning is seen as a collaborative process in which adults and children search for meanings together” (Clark, 2004, p. 143). Informed by these theories and observations, I believe that, through rich interactive learning opportunities, children can develop understandings of mathematical concepts and that play situations can provide such rich learning opportunities.

Mathematics is present in much of children’s spontaneous play (Seo & Ginsburg, 2004). Thinking of play as a disposition, or habit of mind (Carr, 2001), helps to link it with other dispositions that are valued in education, including mathematics education, such as creativity, curiosity, problem posing and problem solving (Ginsburg, 2006; Perry & Dockett, 2010; Moore, Tank, & English, 2018). Perhaps the lack of empirical evidence on the benefits of play for young children is one reason that play is often replaced in early childhood classrooms and homes with more structured activities (Hirsh-Pasek, Golinkoff, & Berk, 2009; Ramani & Brownell, 2013). The mathematical inquiry described in this paper is an attempt to add to the empirical evidence on the benefits of play.

Observing, eliciting and valuing children’s thinking

It is important for teachers and researchers to come to know students’ current and developing understandings, to listen, to respond and to challenge children’s mathematical thinking. In so doing, they help children to construct new knowledge.

Observing children as they explore offers insights into their thinking. The intention in the research contexts reported here was for the adult to intervene as little as possible. The role of the researcher was to observe, listen and interact appropriately to probe children’s thinking and to seek explanations of their actions. Informal conversations were held in a restrained way with the children while they investigated. Thinking conversations were initiated where children were respected as experts in their thinking and learning, and adults questioned and listened to gain information about children’s thinking (Lee, 2012a). The intention was to get children’s perspectives and their reasoning (Dockett & Perry, 2007).

Listening and responding to children involves noticing “when powerful new connections open up for individuals or groups” (Hipkins, 2009, p. 15). As Lansdown (2004) advised, it is important to listen, “to learn to hear and see what children are saying and doing without subjecting it to a filtering process that diminishes their contribution” (p. 5). Together with other researchers, I consider children to be experts in their own lives. When children share their thoughts freely, we achieve a better understanding of their perspectives and competencies (Clark, 2007; Formosinho & Araujo, 2006; McDonough & Sullivan, 2014; Schiller & Einarsdottir, 2009).

Eliciting children’s thinking to encourage mathematical communication is discussed extensively in the literature (e.g., Cheeseman, 2009). More recently, Lee (2012a) noted the importance of using questions that encourage children to talk about their reasoning and their thinking approach. Lee termed this method *thinking conversations* and explained that the purpose is not to find out what children know or have learned or give them information, but rather “to elicit information on their thinking” (p. 7). It is a *thinking conversations* type of approach, which was used in the collection of the data, reported here. Children’s thinking was explored using questions that required higher order thinking and were genuine questions arising naturally in conversation.

Method

The purpose of classroom observation in the research design was to provide an outsider’s perspective of the events. My experience as a primary teacher, teacher educator and mathematics education researcher was used to notice things that would go unnoticed if only video data were collected (Lesh & Lehrer, 2000). As soon as possible after the event, notes were written summarising the session, photographs were taken of relevant children’s work samples/constructions, and any relevant information that would otherwise be lost was documented. These reflections were a record of the first impressions of the events and provided an observer’s perspective. However, I recognise that no perspective has access to ‘the truth’ of a situation.

A *complementary accounts* methodology was used to collect the data reported here. The original research method was described by Clarke (1998, 2001), and an adaptation of complementary accounts method was created by Cheeseman (2010) for use with young children. The intention of the method is to collect data that potentially document different perspectives of the same event. In the cases reported in this paper, video evidence, photographs, the researcher’s view and the child’s view formed the data set. These accounts complemented each other to provide a nuanced depiction of occurrences. The data sets were synthesised to form the ‘stories’ of events as a narrative.

Findings

Three stories will be used to illustrate problem solving persistence of six-year-old children. I define a problem as a situation for which a solution is not immediately apparent (Baroody, 2000), and I acknowledge that problems are solved by effort but also may be solved by play or some creative insight (Bailey, 2017). In the stories recounted here, persistence is taken to mean “how much students keep trying to work out an answer or to understand a problem even when that problem is difficult or is challenging” (Martin, 2003, p. 46). The stories come from sources of data of three separate research projects: *Measurement and Number Sense* (Cheeseman, Benz & Pullen, 2018); *Investigating Children’s Thinking about Measurement Tools* (Cheeseman, McDonough & Golemac, 2017); and *Investigating Early Concepts of Mass* (Cheeseman, McDonough & Ferguson, 2014).

The stories of children’s behaviours were chosen because they describe children of similar ages in different settings and in different countries. While their behaviours are not claimed to be representative, they are chosen from many similar events I have witnessed where children explore and experiment with problems that intrigue them. The order in which the stories appear is determined by the characteristics of the settings in which they occurred: beginning with the most unconstrained free exploration and ending with the most prescribed learning context in a school lesson.

Marni's number tiles

The first is the story of Marni (all names are pseudonyms), and the event took place in a German kindergarten setting. The children had sung songs and played games and they were then free to choose anything they wanted to do inside or outside the kindergarten. When I began watching Marni she had just entered the space that the kindergarten teachers described as the mathematics space. Initially she looked at the equipment on the floor—large plastic tiles that had press-out digits and interlocking edges. Some were already interconnected: a 4 tile was joined to the left of a 3, and a separate piece linked 6 and 7. She began to rearrange the 4 and the 3 to make them read from left to right in the correct order. A boy was playing beside her, rolling on the floor and getting in the way. She wanted him to help but he would not and continued to get in her way. After a minute or two, she stormed off annoyed. This situation is reminiscent of that described by Ramani and Brownell (2013) who found that kindergarten children had some difficulties cooperating with their peers while problem solving during play. For the shared goal to be achieved, children must have a mutual understanding of the task, the final product, and the process to complete the joint goal. It was plain that Marni's playmate did not share her goal; nevertheless, she returned a minute or so later with her hands on her hips and a determined look on her face. Then she returned to her self-defined problem of putting the numbers in sequence, ignoring the boy. With concentration, she systematically gathered the pieces necessary to complete the 2, 3, 4 (Fig. 1), then added 5 and 1 in the correct positions.



Figure 1. Parts of the number sequence coming together.

Marni knew that I was watching her but paid no attention at all to me. Only when she could not locate one of the puzzle pieces that she needed, and I silently offered it to her from underneath a pile of pieces, did she even look up. She simply smiled and accepted the proffered piece, immediately putting it into the correct place. She had set herself a challenge and was intent on completing her task. As she tried to swing the joined numbers into place, the pieces fell apart and wordlessly I helped her by holding one end as she joined the two strips together to form 1, 2, 3, 4, 5, 6, 7, 8, 9. Marni seemed happy until she found the zero and its central 'hole'. The problem then seemed to be, what was she to do with those pieces? She hesitated and paused, and then she added those pieces into the frame that she had placed after the 9 digit (Fig. 2).



Marni placed zero after nine.

Figure 2. Marni’s ordering of the digits 0–9.

She was delighted with herself. She looked happily at the solution to her problem for about 10 seconds. Then she began to take the sequence apart and arrange the numbers vertically. She placed them one by one against the wall from 1 to 9, then zero. She skipped along the number track and sang a song to herself. Then she paused for only seconds and began the next challenge she had in mind. This was to arrange the digits to match the hopscotch pattern on the floor rug (Fig. 3).



Figure 3. Arranging numbers 1–9 to match the rug.

This was indeed a difficult problem because, although she had a clear recognition of where each digit was to go, the edges on each tile did not interlock easily. Here I wordlessly offered to hold and to press the joins with her and she silently accepted the help. The mat had only 1–9 so she had the zero in her hands and was looking puzzled. A teacher came by at just that moment. She asked the child to count

aloud with her and she said the number names one to nine aloud, then looking at the piece the child was holding she said, “Zero”. Then she told Marni to put the zero “down with the one”. With that, Marni carefully put the 0 beside the 1 as shown in Figure 4. Thereby making 10. Whether she had initially seen the zero as a ten is unknown. However, it seems that is what she understood the teacher to mean.



Figure 4. Marni placed the zero beside the 1 to make 10.

The teacher had left and time for play was over as the children began having lunch. Altogether, Marni had been playing with the equipment for 20 minutes, thinking of challenges for herself, solving each problem and setting a new challenge. Analysis of the mathematical content of her problem solving reveals complex reasoning about recognising and interpreting numerical symbols, sequencing digits 1–9, trying to understand zero, building a 10 with the digits 1 and 0, and knowing that the counting sequence 1–9 can be written from left to right and from bottom to top. In addition, there was spatial thinking involved in finding the puzzle pieces to connect the background to the digits and to insert the ‘holes’ in the centre of the digits (e.g., the 4, where the triangular centre piece needed to be correctly oriented). Matching and mapping was also required to place the digits on top of the numbers on the rug. This is all substantial mathematics for a pre-school child, and she posed the challenges and solved all of the mathematical problems herself.

Thomas’ balancing

The second story is of Thomas, an Australian child of six years old, who I observed playing with suspended balances that were novel to him. The context was a play space in a school setting where researchers were investigating children’s spontaneous use of mass equipment (Cheeseman, McDonough, & Golemac, 2017). Pairs of balance scales were set up and objects were laid out for the children to explore uninterrupted. The suspended balances were a simple construction of doweling, bulldog clips, string and plastic pans (Fig. 5). A range of equipment was available with which the children could experiment—small soft toys, commercial cubes (centicubes and unifix), counters, and sets of weights of several different kinds (metal, plastic cuboids and plastic disks). The equipment was available on a large table and the children were invited to explore it. The role of the adults was to observe, to listen and to interact appropriately; to probe the children’s thinking; and to seek explanations of their actions.

Thomas was observed using elements of imaginative play involving the scales. He said, “I’m making this a hot air balloon ride” and he filled a pan with unifix and referred to the cubes as “the people”. A researcher asked him about his play:

Researcher: So what have you found Thomas?

Thomas: This one would go higher and this one would go lower.

Researcher: So what makes it go lower?

Thomas: Because it has got more.

From this reply, we see that, while the experimentation was imaginative, his play was seen by Thomas for the mathematics it was. Van Oers (2014) described productive mathematising as “dialogic, inquisitive, productive thinking” (p. 112). Typically mathematising is driven by personal engagement with a query that requires creativity and endurance. Van Oers linked mathematising to play where mathematics emerges from outside as an attribution of mathematical meaning to children’s actions or utterances by more knowledgeable others. As can be seen in the example of the hot air balloon, the researcher stimulated mathematising by the child. The tools engaged Thomas’ curiosity; he became creative about what he could do and what he could imagine, and the researcher’s questioning led him to think about the meaning of his experimentation. In all, he played with the equipment for more than 20 minutes, changing his successive challenges as he solved each problem he posed for himself. Thomas was very interested in the suspended balance but he found it awkward to hold them up as he filled the scales. He asked a researcher to hold the scales and later invented a way to suspend the balance from a video tripod (Fig. 5). He then spent some time getting the pans in equilibrium. He became fascinated with adding more suspended balances to his display until it began to look like an elaborate mobile.

Thomas showed a range of thinking when using the suspension scales. He did what I have seen many children of 3–6 years old do; he alternately placed objects in each pan and watched the resulting swing of the pans as they seesawed backwards and forwards with the weight. While we do not completely understand this behaviour, we hypothesise that the child is watching and observing the action and reaction as the pans are utilised. Often the child says, “that’s heavy” as the pan goes down and it seems to reinforce the concept that the mass takes the pan lower. Thomas also tried all sorts of combinations of toys in multiples on either side of the balance scales and reported on his findings in terms of comparative weights. For example, he announced, “The fish and the bear are heavier than the duck and the cubes.”



Figure 5. Thomas inventing a stand for his balance.

Having played around with the equipment, he changed his purpose to finding two items that would bring the balance scales to equilibrium. I conjecture that this process is a pattern of exploration that is common to many children who have the opportunity to play with balance scales (Cheeseman, McDonough & Golemac, 2017).

Analysing Thomas’ playful experimentation with novel equipment shows the way he figured out how the balance scale worked by repeatedly testing it and observing the results. He then set himself a new challenge to create a stable platform from which he could investigate equilibrium. Having successfully achieved a point of balance with one set of scales, he elaborated his problem by arranging multiple perfectly balanced scales in a way that was complex, interesting and amusing to him. Thomas increased the challenges he posed for himself as he played.

Violet's weighing

Violet was six years-old and in her Australian Year 1 classroom in her mathematics lesson when the events that I am relating took place. The teacher set up the problem of the day for her children. There were balance pans on every table in the room and the children sat on the mat in a circle. There were bags of carrots, onions, potatoes and oranges and a set of balance scales in the centre of the circle. The teacher explained that the children were to work in pairs; they were to take one of each fruit and vegetable and to arrange them in order from the heaviest to the lightest on their table then to record their findings (Cheeseman & McDonough, 2016).

Soon the children were collecting the fruit and vegetables to weigh and order them. I watched Violet choose her partner and get the carrot, onion, orange and potato. As she picked up each and put it into the crook of her arm, she deliberately held each in turn and hefted it to compare it to the last she had picked up. By the time she had reached her table, Violet had seemingly solved the set problem of the day. She carefully placed the onion then the orange, then the carrot and last the potato in a line on the tabletop. I was wondering what she was going to do for the rest of the hour devoted to mathematics that day. She turned to her partner to ask her what she thought, and together they checked their hefting by systematically using balance scales. They were satisfied and went to find their mathematics books to record what they had done. As Violet settled in her seat, I asked her, "What did you find?" She told me the results from the lightest to the heaviest. Then she said, "Actually I think we have the heaviest carrot in the room!" I asked her if she could prove it. Violet smiled and took the challenge. She asked a child of one pair opposite at the table if she could "borrow" her carrot. She compared the weights by placing one in each pan of a balance and said, "Well it's heavier than theirs." She repeated the process with the other carrot at their table. Then without a moment's hesitation, she left for the adjoining table to continue her testing. At that point, I joined another working pair to observe what they were doing.

As the lesson was nearing its end, I returned to Violet asking her:

- Researcher: So what did you find?
- Violet: Well, actually we didn't have the heaviest carrot in the room. Sammy did. We had the second heaviest carrot in the room.
- Researcher: Really?
- Violet: Yes! And Robert had the heaviest onion, and Sara had the heaviest potato, and Joe had the heaviest orange.
- Researcher: Goodness! Did you test them all?
- Violet: Yes. [Then with a huge, deep sigh.] Now we need to write all about it.
- Researcher: Would you like me to write it while you tell me all about what happened?
- Violet: Yes please, that would be great.

With that, Violet started to describe the whole process: how the question arose and how she and her partner went about answering that question and asking the same question of all of the fruit and vegetables. Finally, we had two pages of findings and she was ready to draw some pictures.

What became clear to me at the time was how determined both Violet and her partner were to solve their problem. The original testing of their carrot would have involved 10 separate weighing comparisons at least. Therefore, in extending the problem to all four fruit and vegetables, the two girls had conducted around 40 systematic tests on their own initiative. They had planned their experiments, conducted them systematically and remembered the results.

While I think that these two girls were unique in their experimentation that day, I also know that there were other children in that class conducting their unique investigations with the equipment they were provided. In fact, the teacher's problem of the day was just the means to launch some very interesting mathematical inquiry that day. The children used their natural curiosity to extend the

teacher's problem into playful learning. These findings are reminiscent of Ginsburg's statement that "children also play with the teacher's mathematics—the lessons taught in school" (2006, p. 145).

Conclusion

The stories recounted here show that children, in this case six-year-olds, are interesting young mathematicians who set and solve their own problems. All they seem to need is an engaging, thought-provoking context to spark their curiosity and set them to problem posing. This echoes earlier findings that it is possible to stimulate curiosity about mathematics by offering children challenging tasks and interesting mathematical tools, listening carefully to be aware of the learning potential children bring to the situation, and encouraging curiosity by showing a real interest in children's investigations (Cheeseman & McDonough, 2016). What we did not discuss in earlier writing was the time and the 'space' children seem to need to investigate ideas that interest them. Here I have described the serious mathematics children explored when they had time to play, problem solve and inquire. Perhaps playful mathematics needs reimagining in pre-school and school settings. Unfortunately, all too often play is replaced in early childhood classrooms and homes with more structured activities to promote young children's academic skills (Ramani & Brownell, 2013). The practice is unfortunate because structured mathematical experiences may deprive children of the joy and fascination of mathematics as well as higher-quality play resulting from their increased mathematical knowledge (Clements & Sarama, 2018).

As Ginsburg and Amit said,

Although young children's thought is different from adults', they deal with mathematical ideas in everyday play, are curious about the subject, know something about it, and can learn interesting mathematics when they are taught. Before the onset of formal schooling, young children do not rely solely on memorization; they do not employ only mechanical skills; they do not operate only on a "concrete" level. They deal spontaneously and sometimes joyfully with mathematical ideas. These are the facts that effective early childhood teaching must recognize and take into account. (2008, p. 275)

Acknowledging that young children are natural inquirers who pose and solve mathematical problems has implications for teaching and learning. Based on the data presented, I suggest that teachers encourage and foster children's problem posing by providing opportunities for children to engage with playful mathematics, planning for plenty of time for children to pose and to solve their own problems, and by watching and listening but intervening only to inspire children's mathematical investigations.

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