**Wilf Malcolm Institute
of Educational Research**
Te Pūtahi Rangahau Mātauranga o Wilf Malcolm

THE UNIVERSITY OF WAIKATO

## Title of Issue/section: Volume 18, Issue 1, 2018

## Guest Editors: Sashi Sharma and Carol Hamilton

## Editor: Noeline Wright

# Can we have 'half children'? Primary in-service teachers' knowledge of division

**Hem Chand Dayal and Govinda Ishwar Lingam**
The University of the South Pacific
Fiji

## Abstract

*Knowledge that teachers bring to the teaching context is one of the key factors in discussions about mathematics teaching. This study aimed to explore in-service primary teachers' knowledge of fraction division using division tasks. The first phase of the study examined fifty-one primary in-service teachers' written responses to division items. This sample of teachers represented seven different countries in the South Pacific region. The second phase of the study used focus group discussions with a smaller sample of teachers, to provide an in-depth analysis of teachers' knowledge of fraction division. The results from phase one indicated that the in-service teachers lacked a sound conceptual understanding of fraction division. In the second phase of the study, however, two out of the five teachers showed signs of developing a conceptual understanding of fraction division. This has implications for teacher professional learning and development.*

## Keywords:

Teacher knowledge, in-service teachers, fraction division

## Introduction

Teachers' mathematical knowledge has become an important area of research in the past two decades. It has been argued that one cannot teach an area of the curriculum effectively if one's own knowledge of it is but limited (Maher & Muir, 2013; National Research Council (NRC), 2000; 2001; Ma, 1999; Risvi & Lawson, 2007; Fennema & Franke, 1992). This proposition is supported by research evidence. For example, Hill, Blunk, Charalambous, Lewis, Phelps, Sleep & Ball (2008) report that stronger teacher knowledge in mathematics yields benefits for classroom instruction and student achievement. Elsewhere researchers have identified different categories of knowledge that a teacher must possess in order to teach a given mathematical concept effectively. For example, seminal work by Shulman (1986) identified three important categories of teacher knowledge: subject matter knowledge, pedagogical content knowledge (PCK), and curricular knowledge. Of particular significance to this study is Shulman's second category, PCK, which he explained as the ability to formulate and represent mathematics in ways that the subject becomes comprehensible to others.

One particular content area in mathematics to receive a growing interest with respect to teaching and teacher knowledge is fractions (Ball, 1990; Simon, 1993; Lamon, 2007; Roche & Clarke, 2013; Chinnappan & Forrester, 2014). Despite this increased attention, the teaching of fractions remains a challenging and problematic area for many teachers (Leung & Carbone, 2013; Chinnappan & Forrester, 2014), and division involving fractions remains one of the least understood topics in primary mathematics (Tirosh, 2000). From the pioneering research on fraction division, such as that of Ball (1990) or Simon (1993), through to the more recent studies of Roche and Clarke (2013) or Chinnappan and Forrester (2014), a persistent finding is that a good number of primary school teachers have a relatively inadequate understanding of fraction division. To bring about improvements in the quality of mathematics teaching, it is important for teachers to have adequate knowledge of the subject matter. Therefore, an investigation into teachers' knowledge of fraction division seemed necessary, given that research on in-service teachers' knowledge of fraction division has been less prevalent (Roche & Clarke, 2013).

The overall aim of this study was to explore primary in-service teachers' knowledge of fraction division. Fraction division in our study is defined as a division that has a divisor of less than one. The following research question guided the first phase of the study: What is the level of in-service primary mathematics teachers' knowledge of fraction division? Following analysis of data from this question, three subsequent questions were posed in the second phase of the study. These included: What level of knowledge of fraction division do teachers exhibit when approaching a division task in groups, and do they make the same conceptual mistakes? If yes, how are these elicited and negotiated in a focus group set-up? How does teacher knowledge transform into an imaginary teaching situation?

In the following section, we present a short conceptual framework for fraction division, followed by a brief review of research literature on teachers' knowledge of fraction division. This is followed by the specifics of the research. The paper concludes with a discussion of findings, conclusion, and a few recommendations for teacher education and research.

## A conceptual framework for understanding division

Two interpretations of division are common (Haylock & Manning, 2014). The first is called the partitioning or the partitive model. The main idea in this model is that of sharing into equal amounts. For example, $20 \div 5 = 4$ could be interpreted as: *I have twenty lollipops and I share it equally among five of my friends. How many lollipops will each friend get?* Another way of looking at the same division algorithm is to ask: *I have 20 lollipops. I want to make smaller packs, with each pack containing five lollipops. How many empty packs do I need?* This second way of conceptualising division is called the measurement, or the quotitive model of division. In the partitive model, I am making five shares of four lollipops in each share. In the quotitive model, I am putting five in one packet, five in the next, five in the third, and the remaining five in the final pack – leaving me with no remainder, and four groups of five. The former consists of five groups of four and the latter has four groups of five. However, this interpretation does not make the difference explicit. The critical distinction here is that in the quotitive model, we are interested in calculating how many packs we will make, whereas, in the partitive model, the divisor (5) already indicates the number of packs (or shares) we are going to make (Roche & Clarke, 2013; Cinnapan and Forrester, 2014).

The measurement, or quotitive model of division, becomes useful when confronted with problems where the divisor is smaller than one, such as a fraction. Consider, for example, a problem used by Roche and Clarke (2013). It reads:

> Solve this: $8 \div 0.5$... Which form of division is most helpful when making sense of
>
> this problem?

Suppose we want to make use of the partitive model here. Sticking with the lollipops, the question to ask is: *I have 8 lollipops and I share it equally among 0.5 of my friends. How many lollipops will each friend get?* Undoubtedly, this is nonsensical. Now, figure the same problem using the quotative

model. A useful way to understand the problem is to ask: *I have 8 lollipops. I want to make packs such that each pack will have 0.5 (half) of a lollipop. How many empty packs do I need?* This statement makes sense and should eventually lead the learner to an answer of 16. This model of division could also be interpreted as repeated subtraction (and repeated addition as well) (Haylock & Manning, 2014). However, when dealing with division where the dividend is smaller than the divisor, repeated subtraction is not useful. The partitive model is the most common method used by teachers as well as children (Ball, 1990; Roche & Clarke, 2013). Over emphasis on this model of division would mean that many students, as well as teachers, would face some difficulties in explaining fraction division. This is because the partitive understanding is insufficient when the division problem has a divisor that is smaller than one. While the quotitive model is helpful in such cases, it requires a more complex level of understanding. The next section looks at teachers' knowledge of fraction division.

## Literature

In her major, formative work on teachers' knowledge of fraction division, Ball (1990) investigated nineteen (ten pre-service elementary and nine secondary) teachers' knowledge of division. She developed and used three items that could be seen as covering the important areas of mathematical knowledge related to division problems. Her first item was called *division with fractions* that required the participants to solve a division algorithm. After the participants tried to solve the division problem, they were asked to write a suitable real-world story that would best represent the division. With respect to the division algorithm, a majority could do the procedure correctly. However, only five could provide a workable representation of the division problem. The findings indicated that while the teachers in her study had procedural knowledge of fraction division, they lacked a conceptual understanding of division.

According to Rittle-Johnson and Siegler (2001), procedural knowledge is the ability to execute rules to solve problems. Conceptual knowledge, on the other hand, requires an implicit or explicit understanding of the concept to be learned. Skemp (2006) used the terms 'instrumental' and 'relational' understanding to distinguish between the two kinds of knowledge. Instrumental understanding involves memorizing rules that work for a given problem. An example of such understanding on fraction division is *to divide a fraction, you turn it upside down and multiply*. On the other hand, relational understanding requires one to question why the particular rule works. Rittle-Johnson and Siegler (2001) claim that both procedural and conceptual knowledge are important and both types of knowledge support each other. Although their findings are based on research with primary school children, they claim that procedural knowledge is useful in improving conceptual knowledge, just as conceptual knowledge is essential in the selection and application of correct procedures. This understanding is important, given that fraction division is one such topic that requires a good mix of both types of knowledge.

The division problem for the current study was taken directly from Ball's prior study, except that it additionally required teachers to draw a model and also write a story problem. However, for this study, the idea of writing a real-world story problem was not made explicit. Another difference between the current study and Ball's is that this study deals with a greater range of in-service teacher samples from five different educational jurisdictions. Whilst the Ball study made use of interviews alone, the current study initially probed teachers' understandings in an examination setting, followed by a more open-ended discussion on a separate teaching occasion.

In another seminal study related to the current study, Simon (1993) asked thirty-three pre-service teachers to write story problems that required working out 51 divided by 4, for which three answers were possible (rounding up, rounding down, and the exact answer). The findings were similar to those of Ball (1990), which was that teachers showed a weak conceptual understanding of division. Of particular relevance to the current study was Simon's (1993) use of a fraction division problem adopted from Ball (1990). The findings indicated that seventy per cent of the participants were unable to write a story problem representing $\frac{3}{4} \div \frac{1}{4}$. These two studies date back more than two decades.

However, similar findings have been noted in more recent studies. A notable point is that the severity of the lack of teacher knowledge seems to have diminished slightly.

Tirosh (2000) explored thirty Israeli pre-service elementary teachers' knowledge of division by fractions. This study provides evidence that teachers could be expected to make the same mistakes as students, for example, inverting both the divisor and the dividend while dividing fractions. The study also noted that these teachers could only imagine procedural errors that students could make on the division problems: they could not describe any intuitively based mistakes that students would be expected to make. The study found that most teachers were aware of the limitations of the partitive model of division.

In a recent study based in Australia, Roche and Clarke (2013) investigated practising primary teachers' mathematical knowledge of the two models of division. The division tasks used in this study were similar in that participants were asked to draw pictures to represent whole number division ($12 \div 3$) and write an appropriate story problem. Less than half of the 378 teachers gave correct representations of the two models of division on the post-test, despite some professional learning intervention. The results, however, indicated that more teachers got the correct representations on the post-test, indicating that teacher knowledge had developed during the intervention period. In a final division problem, where the divisor was a fraction ($8 \div 0.5$), a majority of the participants were able to provide the correct answer. The common misconception was that of dividing by 2 instead of 0.5. Leung and Carbone (2013) investigated Hong Kong pre-service teachers' understandings of fraction division by asking them to present real-life story problems. Findings from an analysis of teacher posed story problems revealed a lack of understanding about which model is best to represent fraction division problems.

In another Australian study, Chinnappan and Forrester (2014) investigated pre-service teachers' representation of fractions. Of the four division tasks, the final one was related to the current investigation. This division algorithm ($1\frac{1}{2} \div \frac{1}{4}$) was well solved by the pre-service teachers, showing that a majority of these participants demonstrated a solid procedural, as well as conceptual knowledge, of division. In their solution, participants did draw from the correct (measurement) model of division. In another Australian study, Although the literature reviewed above has noted similar findings on teacher knowledge of fraction division, each of the studies had its own limitations. Given the nature of the studies, only a few of them have large sample sizes. Most, except that of Roche and Clarke (2013) sampled pre-service teachers. There seems to be a dearth of literature on comparative groups of participants from different teaching contexts. This study hopes to contribute to our understanding in this regard.

This study therefore, addresses a gap in research on teachers' mathematical knowledge of fraction division from a South Pacific context, a region in which research of such a nature is relatively sparse. Exploration of how practising primary teachers would solve division algorithms involving fractions, where both the dividend and the divisor were fractions, plus how they would represent such division problems by using models or stories would help better understand their knowledge of fractions and would assist in recognising suitable solutions to the problem. The purpose of the study, therefore, was to answer the principal research question: What is the level of in-service primary mathematics teachers' knowledge of fraction division. This study made use of division tasks designed by Ball (1990) on a small cross-section of primary in-service teachers, from seven different countries in the South Pacific region. The initial phase of the study made use of teachers' written responses to three division items. A smaller sample of teachers was then given the same task in a focus group set-up. The study's sample involves fifty-one in-service primary teachers. This choice of sample is relevant given that research on division by fractions involving in-service teachers has not been widespread (Roche & Clarke, 2013). More details of the participants and method adopted are provided under the next section.

## Method

The study proceeded in two phases. Because we wanted to explore the knowledge of fraction division of a diverse sample of in-service primary teachers, as well as provide an in-depth account of how a small sample of these teachers featured on the same problems when given a second chance at it, a mixed methods research approach was adopted. According to Punch and Oancea (2014), mixed methods research is that which requires a combination of two or more types of data and different methods of collecting these data. Phase one of the study utilised a non-experimental test design. This research strategy allowed us to identify the quality of knowledge of fraction division among a diverse group of primary in-service teachers. In the second phase, we used an interpretive research design to further explore participants' knowledge of fraction division. The participant and instrument for each phase of the study are discussed in this section.

### Phase one

The participants of the first phase of this study consisted of 51 in-service teachers who were enrolled in a mathematics education course at The University of the South Pacific. The participants represented seven different island nations. The sample consisted of 27 females and 24 males. All the teachers in this sample had at least a Diploma in Education from their respective national teacher colleges, as well as a minimum of two years of teaching experience. A summary of the number of participants from each country for phase one of the study is provided in Table 1.

**Table 1.    Phase one participants**

| Fiji | Vanuatu | Tonga | Marshall Islands | Solomon Islands | Kiribati | Tuvalu |
|------|---------|-------|------------------|-----------------|----------|--------|
| 27 | 1 | 3 | 11 | 5 | 3 | 1 |

The first phase of the study utilised fraction items taken directly from prior research (Ball, 1990; see Appendix 2). The items were judged to be appropriate, given that these teachers are experienced practitioners and would be expected to read and interpret the task accurately. The mathematics education course was offered via a distance and flexible mode. Therefore, using a test format was judged to be a viable method to elicit understanding of fraction division across a wide range of participants.

### Phase two

The second phase aimed to re-examine the fraction division knowledge of a small group of teachers. One of the limitations of phase one procedure was that a formal test situation could have resulted in teachers providing responses to gain marks. This could have prevented them from thinking 'outside the box'. Another limitation of a formal written test could have been a lack of time. Based on these, and other possible limitations of formal tests, such as anxiety or fear, the researchers decided on further exploration of teachers' knowledge of fraction division.

The selection of this group of teachers was entirely non-random. Two weeks after the mid-semester test, the researchers organised a face-to-face workshop for students based at the Suva campus. The first hour of the workshop was devoted to the three division items. Only five teachers turned up during the start of the workshop session. They were divided into two groups, group one of three teachers, and group two of two. The five teachers who formed the sample for the second phase were from Kiribati (3), Tonga (1), and Solomon Islands (1). They were all based at the Laucala Campus in Suva. All of them agreed to be part of this study. With the permission of the participants, the conversations were digitally recorded to maintain accuracy. The transcribed data were subjected to

qualitative analysis using the framework for analysis that was used in the first phase (see Appendix 2). However, for this explorative phase, we gave more attention to detail to what correct and incorrect information each participant made explicit. The participants had generally failed to display a conceptual understanding of division by fractions in their written responses to the test item. Detailed participant information is summarised in Table 3 (see Appendix 1). Participants' real names are withheld; instead, pseudonyms are used.
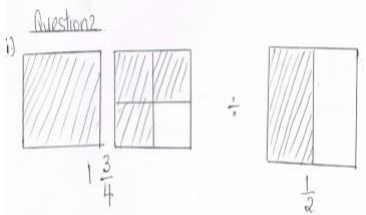
## Results

The results from each phase of the study were analysed using the framework (see Table 4 as Appendix 2). Each response was read by both the authors against the criteria given in the framework for analysing results. The findings from each phase are presented separately.

### Phase one

The findings of phase one are succinctly represented in Table 2, on the basis of each item in the examination. As each examination script was read, it was assigned a numerical value starting from one up to fifty-one. These numbers are represented as the identities of the participants' quotes in the discussion as their actual names were kept confidential.

**Table 2.   Findings from phase one**

| Item | Summary of findings | Examples of teacher responses |
|---|---|---|
| Teachers' knowledge of fraction division:<br><br>$1\frac{3}{4} \div \frac{1}{2}$ | 33 out of 51 teachers (65%) showed use of a correct procedure. Six out of 51 teachers (11%) had followed a correct procedure but had minor computational errors. | A common method of solving such division problems involves changing the mixed number into improper fractions, inverting the divisor and completing the multiplication procedure (Participant 20). |
| | Twelve out of 51 teachers did not demonstrate any procedural understanding of solving this division problem. | Work out the fractions and multiply the answer with the whole number (Participant 24).<br><br>Change the fractions into decimals before doing the division (Participant 30). |
| Representing fraction division | None of the teachers were able to come up with a correct model or diagrammatic representation.<br>Thirty nine out of 51 teachers (76%) did not provide any representation. | Example of an incomplete response<br> |
| Story problems | Four out of 51 teachers (8%) were able to write a story. | Example of a correct response: How many $\frac{1}{2}$ litre bottles can be filled exactly from a bottle of $1\frac{3}{4}$ |

Forty seven out of 51 teachers (92%) could not provide a story that matched the fraction division.

litres of juice. Do not round off your answer. (Participant 17).

Example of an incorrect response: Mary had an orange. Her mother gave her another $\frac{3}{4}$ of an orange.

Mary then shared the oranges equally between her two brothers. What fraction of the orange did each brother have? (Participant 18).

## Phase two

As mentioned earlier, Phase two of the study involved only five teachers: Tui and Lyn as group one and Kabu, Claire and Sarah as group two. The way in which the teachers in the two focus groups went about making sense of fraction division is presented next.

## Focus group one

This pair (Tui and Lyn) began their discussion by asking questions of each other about how to explain the division problem to their learners. They were able to work out the division algorithm correctly by changing the dividend to an improper fraction and multiplying it with the reciprocal of the divisor. The pair showed no signs of difficulty with the algorithm. The pair quickly moved on to discuss reasons behind the operations. The following transcribed conversation was centred on finding ways to make sense of the procedural calculations. The pair was unable to find a convincing reason for inverting and multiplying.

Tui: Suppose one of the students asks why do you invert and multiply. Why do we change? What will your answer be to that?

Lyn: Yes, I agree. We should be able to explain this. But what did we learn in school? Just invert and multiply.

Tui: The answer is 3 whole and a half remaining.

Lyn: Why do we reverse and multiply?

Tui: Why don't we reverse the other one?

Tui: It is not enough to say change and multiply. We teach what we learn from our teachers.

Tui: Do we change both or just one?

Lyn: No…no…only the second fraction.

Researcher: Why do you change and multiply?

Unable to defend this, the pair moved to convert the problem into division using decimals. This, they found easier to conceptualise.

Lyn: I do not know.

Tui: How about we change the question into decimal. For this one, it is 0.5. For this one, it is 1.75. It is easier to divide it this way (Pair carry out long division algorithm and come up with 3.5)

Lyn: It is same.

Tui: Yes. It is easy to explain division in this way because we don't use invert and multiply here. In decimals, we don't invert and multiply. It tells us the relationship between fractions and decimals. It tells us about division and is similar to dividing whole numbers.

With this discussion, the pair went into representing the fraction division using diagrams. This began with Tui explaining that the question required figuring out how many halves there were in one and three quarters. This way of explaining showed some understanding of the quotitive model of division. The pair explained this using numbers. The following drawing was taken from the pair's discussion paper.

**Figure 1.    Fraction division using diagrams**
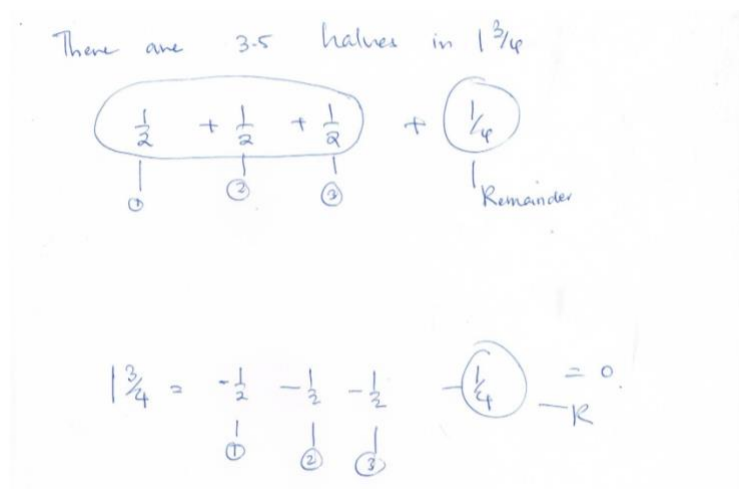


This representation shows that the pair was able to conceptualise division by fractions not only using the quotitive model but also repeated subtraction and addition. The pair, however, were unable to interpret the remainder correctly. Diagrammatically, the remainder is a quarter as shown in the pair's representation. However, numerically, this remainder is half and not a quarter.

When the pair went to the final part of the discussion, which was related to writing story problems representing division by half, the conceptual understanding shown above was not evident. The discussion revealed that Tui had a better understanding of division by half. This was noted when he reiterated his quotitive reasoning model:

We take something from here and give it out.

Sometime later, he said,

We can't share it to half people.

Lyn, on the other hand, showed signs of measurement model of reasoning. She said:

We have a melon and a three quarter and it was eaten by two people.

By the end of the one-hour session, Tui had shown his understanding using a relevant story:

We have one and a three-quarter apples. We cut parts which are half in size and give it away to people. How many parts can we make?

Lyn, on the other hand, was not sure about the correctness of Tui's story, and could not come up with her own story. Her idea of division by half was similar to what she had written in her examination script – dividing by half of the children. Realising that half could not represent the number of children, as pointed out by Tui, she was stuck with division by two. Tui who had mentioned dividing among half the class in his test response was able to conceptualise division by half correctly. Even his test response was not among half children, as reflected by Lyn's understanding. It could be

observed that Tui utilised the repeated subtraction model in making sense of division by half. The idea of giving it out was repeated in his utterances.

## Focus group two

This group of teachers (Kabu, Claire and Sarah) began by reading the questions and writing down the division problem $1\frac{3}{4} \div \frac{1}{2}$ on a piece of paper. The group had no problems in carrying out this division algorithm. A lack of conceptual understanding of the algorithm was noted when Sarah said that this could be done by changing half $\left(\frac{1}{2}\right)$ into two quarters $\left(\frac{2}{4}\right)$. This was probably because she saw $1\frac{3}{4}$ to be equal to $\frac{7}{4}$ and presumed that changing to a common denominator would be helpful. She finally realised that there was no difference in the answer. Sarah had tried to use the same incorrect approach in her test. Claire suggested division using decimals ($1.75 \div 0.5$) but this division algorithm was not pursued. In summary, the group showed a reasonable understanding of the division algorithm, and they could provide some justification of this:

> Sarah: So, we can multiply.
>
> Claire: We need to change to improper fraction (referring to the dividend).
>
> Sarah: We need to find the common denominator.
>
> Kabu: No, we multiply straight.
>
> Claire: So, we invert this (indicating reciprocating half). So that means the answer is $3\frac{1}{2}$.
>
> Researcher: Why did you put the second fraction upside down?
>
> Claire: Because the formula is (probably meaning the rule is), when we change the division sign into times, we have to take the reciprocal.
>
> Researcher: Why? What if one student asks you why?
>
> Sarah: Because we are doing the reverse of division, which is multiplication. So, we also reverse half and make it two over one.

For the second task, the group drew the divisor as two quarters and $1\frac{3}{4}$ was represented as seven pieces of quarter, represented by one whole (four quarters) and another whole with a missing quarter. The group was initially able to represent the division problem by placing the half (divisor) into the dividend. This is shown in Kabu's commentary:

See how many times the half (picture 2) goes inside and fits in here (picture 1). Take this half and place it here, another goes here, then another here. It goes three times and we have small piece left. This is the remainder.

While all three agreed with this explanation, there was no discussion on why the leftover in the diagram was one quarter and how it related to the numerical answer of $3\frac{1}{2}$. This discussion would have indicated how teachers would conceptualise the remainder, in this case, half of a half. Sarah and Claire, however, showed that they did not fully understand the diagrammatic representation given by Kabu. The following conversation revealed this.

> Researcher: How can you write a story problem about this fraction division?
>
> Sarah: Yes, how can we divide?
>
> Sarah: How many children can share this, one and three-quarter pies?

Researcher: Since you said 'share' and 'children', which fraction represents the pie and which fraction represents the children?

Sarah: Let's say, four children?

Researcher: Which number represents four children?

Claire: Okay…how many pieces….

Sarah: The one and three-quarters represents children?

Researcher: The first fraction or the second one represents the number of children?

Sarah: The first one.

Researcher: But you said that one and three-quarters is the pie.

Sarah: The second fraction represents children.

Researcher: How many children do we have then?

Claire: half…

Researcher: Can we have 'half' children?

Group: No (and everyone laughs).

From here on, Sarah and Claire had explicitly stated on more than three occasions that this was a very difficult problem for them. Despite realising that half cannot represent the number of children, the two continued to conceptualise division using the primitive partitive model. The two came up with more wrong examples. For example. Sarah said, How about if we total this up and share with the amount (meaning number of) of children? We have thirteen quarters. The use of thirteen-quarters shows that she was seeing $1\frac{3}{4}$ as $\frac{13}{4}$. This simple mistake was not evident at the start of the discussion

and could probably indicate that Sarah was under some sort of pressure to find an explanation for this fraction division. When asked about a simple division problem ($8 \div 4 = 2$), she said how many apples four children will have? Sarah was having difficulties in moving away from conceptualising the divisor as the 'number of children'. Similarly, Claire was unable to single out which division model would be helpful. At one stage, while making sense of $8 \div 4 = 2$, she said: you have 8 apples and four children. How many shares can you make? How many children will have each share? Kabu, on the other hand, gave hints on three occasions, such as what about if children need half a share? This one here is a child. He should have half a share. We try to bring the half into the whole, we see that half goes inside three times but a quarter remains. This hint was not followed by Sarah and Claire, but they continued to write story problems. Each time they wrote a story problem, the only change we noticed was the choice of the quantity to be shared, alternating from pies, apples, watermelons and cakes. Finally, after repeated aid by Kabu, the two wrote their own story problems. The following discussion showed that Sarah was showing some signs of conceptualising division by half. Claire, however, was unable to show a sufficient amount of understanding about division by half using the  model. Claire continued to see division as sharing where each person gets some amount after the division operation. This was despite almost an hour of focus group discussions.

Kabu: The one here is a child. He should have half, the other one half, and so on.

Sarah: Mere had one and three-quarter apples, and she wants to share among….okay…okay (realises that she may be derailing)…okay she shares half amount to how many friends….like half to one friend, half to another friend….(still unable to put it in a proper question form).

Claire: How many halves will each friend get (joins in but provides an inaccurate question)?

Sarah: How does a child share half of apple if he got one and three-quarter apples?

Claire: Yeah that's good.

Sarah: How many pieces I could cut from sharing half?

Both Claire and Sarah seemed to be confused with sharing 'in equal amounts of half' and 'sharing half'. They were stuck with the idea of sharing half. This could be interpreted as halving one and three-quarters ($1\frac{3}{4} \div 2$), followed by sharing one of the halves. Such an interpretation is not useful in making sense of division by half.

## Discussion

The overall aim of this study was to explore in-service primary teachers' knowledge of fraction division. Findings combined from both phases tend to point to a procedural understanding of the division algorithm with a lack of conceptual understanding of division by half. Such findings were confirmed more than three decades ago (Ball, 1990). In reference to Ball's study, only a few mathematics major participants were able to generate appropriate representations of division by half. Also, similar findings have been noted in more recent studies such as Roche and Clarke (2013) and Chinnappan and Forrester (2014).

The current study also noted that teachers like Sarah and Claire spoke openly about the difficulty with fraction division. Teachers such as Sarah, Claire and Lyn continued to show a lack of conceptual understanding of division by half. Their responses were no different from what they had written weeks ago in a written test. It could be noted that these three teachers, like many others in phase one, only held an understanding of the partitive model of division. Findings from the study seem to suggest that the partitive model of division does not support teacher learning related to fraction division. On the contrary, we found some evidence that the partitive understanding of division hinders teacher learning on fraction division. For example, Sarah had difficulties in moving away from viewing the divisor as the number of children, something she would have inherited from the partitive model of division. The findings also tentatively demonstrate, as in the case of Lyn, that teachers who are able to do a procedure on fraction division would not necessarily be able to explain it to someone else – a key aspect of PCK under Shulman's (1986) teacher knowledge criteria. In our study, we found a lack evidence that supports the claim by Rittle–Johnson and Siegler (2001) that procedural understanding contributes to the development of conceptual understanding, although we can see that understand that this needs to be further explored with respect to teacher learning.

The findings further confirm that teachers have great difficulties when tasked with creating stories that would match the fraction division, as revealed in previous studies such as Roche and Clarke (2013). This difficulty could be attributed, to a certain degree, to language problems. In phase two of the study, for example, Sarah and Claire had problems in understanding 'sharing in equal amounts' and 'sharing half', indicating that they may have been confused with the language of division. These teachers would have difficulties in explaining the fraction division concept to their classes, given that a majority of the upper primary classes in the Pacific use English as the medium of instruction.

The study also notes two examples of success, as seen in Kabu's and Tui's thinking. Both of these participants had shown no understanding of division by half in their test responses. The focus group discussions revealed that they could divide by half. Both of them used 'giving away' or 'giving out' to make sense of fraction division. It could be argued that their representations would have emerged as a result of taking part in the group discussions. This is because neither Kabu nor Tui showed any signs of a correct representation at the earlier stages of group discussion. They tried their best to make sense of division by half by asking questions such as how many times will 0.5 go into 1.75. Kabu utilised decimals to make sense in this way. For the purpose of this study, such an understanding is considered sufficient. This study, however, did not focus upon how the teachers could make sense of the remainder. This is one area that could be explored in future studies. Additionally, the findings from the focus groups must be interpreted in light of the fact that no explicit instruction was given regarding

different models of division. The qualitative improvements in in-service teachers' understanding noted in Kabu and Tui's case cannot, therefore, be attributed to teaching intervention by the researchers.

## Conclusions, limitations and implications

One of the limitations of this study was the small sample of teachers in the second phase of our study. Another limitation was linked to the language of communication during the focus group. Because we had teachers from different cultural and language backgrounds, we had to rely on English as the medium of discussion. Because of this, we were unable to tap into the conceptual knowledge that our participants would have held in their mother tongues (Rittle–Johnson & Siegler, 2001).

On the basis of findings from phase one of our study, it can be stated that primary in-service teachers amongst this study have a limited conceptual understanding of fraction division, as revealed by their inability to provide a logical representation, including a lack of appropriate stories to represent the fraction division. However, the participants were able to demonstrate a procedural understanding of fraction division that included the traditional 'invert and multiply' approach. While procedural understanding is useful (Rittle–Johnson & Siegler, 2001; Skemp, 2006), this limited amount of knowledge of fraction division would be judged insufficient in terms of the pedagogical content knowledge required to teach fraction division (Shulman, 1986).

Findings from phase two revealed that teachers could modify their thinking when given an opportunity to discuss mathematical items in focus groups. This is an important contribution of this study. This assertion also calls for more studies in the future that rely on teachers themselves to modify their thinking in teaching contexts, such as the one highlighted in this study. How this development happens could be another area for exploration. Another area worth researching further would be following these in-service teachers, including the likes of Sarah, Claire and Lyn, in real classrooms and observing their teacher knowledge in a teaching and learning context. The researchers also found tentative support for the use of focus groups in research on teacher knowledge. For example, both Kabu and Tui could provide useful hints to their group members whenever the group members got stuck.

Some of our participants continued to make the same conceptual mistakes. This indicates that fraction division is one area that needs more attention in initial teacher preparation as well as in professional teacher learning programmes and future research. In terms of professional learning and research, we argue that focus group learning could be effective if teachers are allowed to engage in discussions using their own language, given that fraction division is based on understanding the language of division. Such discussions would reveal whether a lack of knowledge of fraction division is due to a lack of mathematical knowledge or is it because of implicit misunderstandings arising as a result of language difficulties. The interplay, if any, between teachers' procedural knowledge and conceptual understanding, needs also to be given due consideration.

## References

Ball, D. L. (1990). Prospective elementary and secondary teachers' understanding of division, *Journal for Research in Mathematics Education, 21* (2), 132-144.

Chinnapan, M., & Forrester, T. (2014). Generating procedural and conceptual knowledge of fractions by pre-service teachers. *Mathematics Education Research Journal, 26* (4), 871-896.

Fennema, E., & Franke, M. L. (1992). Teachers knowledge and its impact. In: D.A. Grouws (Ed) *Handbook of  research on mathematics teaching and learning* (pp. 147 – 164). New York, Macmillan.

Haylock, D., & Manning, R. (2014). *Mathematics explained for primary teachers*. London: Sage.

Hill, H. C., Blunk, M. L., Charalambous, C. Y., Lewis, J. M., Phelps, G. C., Sleep, L. & Ball, D.  L. (2008). Mathematical knowledge and the mathematical quality of instruction: An

exploratory study. *Cognition & Instruction, 26* (4), 430 -551. Doi:10.1080/07370000802177235

Lamon, S. J. (2007). Rational numbers and proportional reasoning: Toward a theoretical framework for research, in F. K. Lester, Jr. (Ed), *Second handbook of research on mathematics teaching and learning* (pp. 629 - 668). Charlotte, NC: Information Age Publishing.

Leung, I. K. C. & Carbone, R. E. (2013). Pre-service teachers' knowledge about fraction division reflected through problem solving. *The Mathematics Educator, 14*(1), 80-92.

Ma, L. (1999). *Knowing and teaching elementary mathematics. Teachers understanding of fundamental mathematics in China and the United States*. Mahwah, NJ: Erlbaum.

Maher, N., & Muir, T. (2013). "I know I have to put down a zero, but I'm not sure why": Exploring the link between pre-service teachers' content and pedagogical content knowledge. *Mathematics Teacher Education & Development, 15* (1), 72-86.

National Research Council. (2000). *How people learn: Brain, mind, experience, and school*. Washington, DC: National Academies Press.

National Research Council. 2001. *Adding it up: Helping children learn mathematics*. Washington, DC: The National Academies Press. https://doi.org/10.17226/9822.

Punch, K. F., & Oancea, A. (2014). *Introduction to research methods*. London: Sage.

Rittle-Johnson, B., & Siegler, R. S. (2001). Developing conceptual understanding and procedural skill in mathematics: An iterative process. *American Psychological Journal of Educational Psychology, 93* (2), 346-362.

Rizvi, N. F., & Lawson, M. J. (2007). Prospective teachers' knowledge: Concept of division. *International Education Journal, 8* (2), 377-392.

Roche, A. & Clarke, D. M. (2013). Primary teachers' representations of division: assessing mathematical knowledge that has pedagogical potential. *Mathematics Education Research Journal, 25* (2), 257-278.

Shulman, L.S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15* (2), 4-14.

Simon, M. A. (1993). Prospective elementary teachers' knowledge of division. *Journal for Research in Mathematics Education, 24* (3), 233-254.

Skemp, R. R. (2006). Relational understanding and instrumental understanding. *Mathematics teaching in the middle school, 12*(2), 88-95.

Tirosh, D. (2000). Enhancing teachers' knowledge of children's conceptions: The case of division of fractions. *Journal for Research in Mathematics Education, 31*(1), 5 - 25.

## Appendix 1

**Table 3: Focus group participants: Bio data derived from one to one interviews**

| Focus Group | Names | Background | Analysis of test results |
|---|---|---|---|
| One | Tui | A Tongan male with 15 years of teaching experience. He had a Diploma in Education from Tonga Institute of Education. Tui failed high school Mathematics. | Tui converted $1\frac{3}{4}$ into 1.75 and $\frac{1}{2}$ into 0.5 but was unable to carry out the long division algorithm using the two decimals. He was unable to draw any diagrams. He wrote a story problem which read: *I have one complete cake and a $\frac{3}{4}$ and I share them out to one half of my class.* |
| | Lyn | An I-Kiribati female with 7 years of teaching experience. She had a Certificate in Teaching from Kiribati Teachers College. Lyn acknowledged | Lyn had procedural knowledge of division by fractions where she 'inverted and multiplied' to solve the problem. She did not show any |

| | | | |
|---|---|---|---|
| | | that she was not good in high school mathematics. | diagrammatic representation. She wrote a story problem which was incorrect: $1\frac{3}{4}$ *of a pie is divided by* $\frac{1}{2}$ *of the children*. |
| Two | Kabu | An I-Kiribati male with a teaching experience of 5 years. He had a Diploma in Education from Kiribati Teachers College. He said that he used to fail mathematics in high school. | Kabu had good procedural knowledge of division by fractions. However, he did not give any representation or story. |
| | Claire | A female I-Kiribati with 7 years of teaching experience and had a Diploma in Education from Kiribati Teachers College. She claimed to be an average mathematics student in high school. | She had good procedural knowledge of division by fractions. She did not provide any representation. She wrote a story problem which was incomplete and incorrect: *A bottle of water contains* $1\frac{3}{4}$ *ml, then* $\frac{1}{2}$ *litres of bottled water*. |
| | Sarah | A female teacher from Solomon Islands, had taught for 9 years. She had a Diploma in Education from Solomon Islands College and rated herself as "good enough" in high school mathematics. | Sarah was unable to solve $1\frac{3}{4} \div \frac{1}{2}$. She found a common denominator of 8 and multiplied the numerators. She may have confused this with addition of fractions. She gave no representation and an incomplete story which read: *Mary had one whole cake and three quarters. Then she wants to divide among...* |

## Appendix 2
## Table 4: Items and framework for analysing response to each item

| | |
|---|---|
| People have different approaches to solving problems involving division with fractions. How would you solve this one? $1\frac{3}{4} \div \frac{1}{2}$ | No evidence of procedural understanding, unable to invert the divisor and perform the correct multiplication, or does not provide an answer at all, or provides an incorrect/partially correct answer. Evidence of procedural knowledge with correct computations. |
| Draw a diagram or a model that will help your students visualise this. | No conceptual understanding of representing fractions or an insufficient representation. Shows evidence of conceptual understanding of division of fractions. Provides a true pictorial representation. Shows an understanding of quotative model or division as repeated subtraction. For example, |

1    3/4         To be divided into
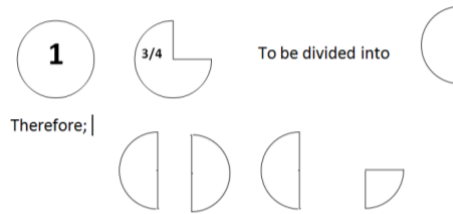
Therefore; |

| Write a story problem for which the above division will form the appropriate mathematical formulation. | Unable to provide a written (story) representation or provides a story that is partially complete or irrelevant. |
| --- | --- |
| | Provides a story that is a true depiction of the division by fraction problem. For example, Ken has one and three quarter 'pizza'. He makes small packs, each containing half a pizza. How many packs can he make? |