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Developing algebraic understanding through talk and writing: A pilot study

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ABSTRACT

This article reports on what happened when a mathematics teacher at an urban, New Zealand secondary school trialled the use of oral and written language tasks with his Year 9 mathematics students (two classes) as a way of developing their understanding of algebra. Over a period of five weeks, he trialled the use of a range of activities that were new to his practice and were designed to encourage students to use oral and written language to express their algebraic understanding. A range of data was collected in relation to this "intervention"—pre- and post-intervention questionnaires, test results and teacher observations. Findings suggested that the activities collectively contributed to a marked increase in students' algebraic confidence and willingness to use algebraic discourse in expressing and reflecting on their learning. It is argued that these findings have implications for mathematics instruction and, more generally, for disciplinary literacy theory.

Keywords

Algebra; algebraic literacy; disciplinary literacy; mathematics discourse

Introduction

This article reports on what happened when the second author, a Mathematics teacher at an urban secondary school in Auckland, decided to trial a range of oral and written tasks with his Year 9 Mathematics class during their introduction to algebra. His thesis was that the use of these tasks would facilitate algebraic understanding and knowledge retention and further, would motivate students to engage with this aspect of Mathematics. Two voices are present in this article, Sam's as teacher researcher and Terry's as critical friend and co-researcher.

Sam had participated in a six-day writing workshop, conducted in January 2014 by the first author (Terry). Colleagues of Sam's from a range of disciplines (subject areas) had previously undergone intensive writing workshop professional learning in 2013 (as Group 1 teachers) as part of a two-year,

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participatory action research project being undertaken in his school entitled: "A culture of writing: Impacting on teacher and student performance across the curriculum" (for an overview see Locke & Hawthorne, 2016 A number of these teachers had already undertaken practitioner inquiry in their own classrooms (e.g. Hawthorne, Locke, & Tai, 2015). Sam was one of a "second wave" of Group 2 teachers who joined Group 1 teachers in a professional learning community (PLC) at the school, aimed at fostering effective writing instructional practices across a range of curriculum areas (McLaughlin & Talbert 2006; DuFour 2004). Sam was the only mathematics teacher in this PLC.

Through participation in the writing workshop, Sam realised that though he had enjoyed writing in primary school, he had found himself struggling with the technical side of writing at secondary school and had begun to think of himself as a poor writer. However, the workshop had reminded him of the importance of writing as a vehicle for expressing ideas. As teacher-in-charge of junior Mathematics in his school, he felt that he could take a lead with his syndicate by himself engaging in professional inquiry with two of his Year 9 (12- to 13-year-olds) Mathematics classes. Encouraged and supported by other members of the PLC, he developed an intervention that would have his students:

- write down their thinking about algebra;
- answer problems using explanations in writing;
- translate algebra into words;
- translate words into algebra; and
- write mathematically using conventional working layout.

His hypothesis was that designing and implementing activities that engaged students in focused talk and writing would help them both clarify and formalise their thinking and lead to a deeper and more robust understanding of the topic. With this in mind he anticipated that the use of peer response (in pairs and groups) would drive and support learning in the classroom. The research questions driving the inquiry were:

1. How do the students view certain planned learning activities in terms of both motivation and facilitating learning?
2. Is there evidence that the planned activities helped students develop and retain mathematical understanding?
3. Is there evidence that students' attitudes to learning algebra changed over the course of the intervention?

Mathematics discourse as disciplinary literacy

School subjects are "recontextualised" (Bernstein 2000) versions of beyond-school disciplines, defined by Young and Muller (2010) as follows:

> All disciplines, in order to be disciplines, have shared objects of study, and in order to be robust and stable, display objectivity—that is to say, they possess legitimate, shared and stably reliable means for generating truth…. Truth is, by this account, a stable relationship between the objects of study and an informed community of practitioners. (p. 21)

In such terms, "truth" is not universal, but rather relational and provisional. In varying ways, school subjects mirror beyond-school disciplines by making meaning in discipline-specific ways using a range of representational resources. These ways of making meaning can be thought of as *disciplinary literacies*—socially constructed, cognitive, technologically mediated ways of making meaning using a range of symbolic (semiotic) systems. Recent reforms in mathematics and science education have prompted an investigation into how discipline-specific literacy is constituted, the kinds of curriculums and classroom practices that best foster disciplinary literacy and "the pedagogical content knowledge, school culture, and instructional approaches needed by teachers in the content areas to achieve disciplinary literacy for their students" (Norton-Meier, Tippett, Hand, & Yore 2010, p. 118)

A question for mathematics teachers (and others) is: How might disciplinary thinking, particularly higher-level thinking, be developed in students via a focus on language—spoken and written—as a vehicle for constructing and transforming knowledge. According to Grimberg and Hand (2009),

> A guiding principle of science education is that engaging in the canonical discourse—meaning language, genre, textual forms, and the use of symbolic conventions—of a discipline is critical for science learning … and therefore should be explicitly taught in the classroom. (p. 504)

The same applies to mathematics understanding. Morgan, Craig, Schuette and Wagner (2014) draw attention to "the central role language plays in the learning, teaching and doing of mathematics", but express a concern about a lack of attention in the research to "the development of the linguistic competences and knowledge required for participation in mathematical practices" (p. 843). In their review of best practice in mathematics instruction, Anthony and Walshaw (2007) note that "Mathematical language involves more than vocabulary and technical usage; it encompasses the ways that expert and novice mathematicians use language to explain and to justify concepts" (p. 2). They also refer to the importance of pedagogical tasks and activities, which develop certain "habits of mind" (p. 3) including the habits of mind of teachers themselves. The understanding of the term *language* we brought to this pilot study was: the application of one or more semiotic modes as resources for expression, representation and communication. From this perspective, mathematical discourse was viewed characteristically as *multi-semiotic* (Morgan et al., 2014, p. 846) and multimodal.

Metalinguistic issues are pertinent to all teachers, because metalanguage is the means of talking *about* the language practices of various disciplinary literacies and therefore an essential tool for metacognition. Though Turkan, De Oliveira, Lee and Phelps (2014) are referring specifically to the teaching of English language learners (ELLs), their definition of disciplinary linguistic knowledge (DLK) as "teachers' knowledge of a particular disciplinary discourse [which] involves knowledge for (a) identifying linguistic features of the disciplinary discourse and (b) modelling … how to communicate meaning in the discipline and engaging [learners] in using the language of the discipline orally or in writing" (p. 2), is relevant to all subject teachers.

As Figure 1 indicates, there is a complex relationship between the technical language (terminology related to content) that is integral to a subject or discipline (e.g. "variable", "factor", "simplify" for mathematics), and the metalanguage required to think about the demands of a particular genre (e.g. "theorem", "justify", "concept"). This distinction relates to Bereiter and Scardamalia's (1987) terms: *content knowledge base* and *rhetorical knowledge base.* A teacher's professional subject knowledge feeds their subject-related pedagogy supported by knowledge of and experience in a range of educational practices. Likewise, their professional knowledge of disciplinary literacy practices feeds their subject-related, literacy pedagogy. The subject-related aspects of professional knowledge (at the top of the figure) and the disciplinary literacy aspects (at the bottom of the figure) are mutually constitutive. That is because the constitution of a body of knowledge exists in a symbiotic relationship with the means used to construct and express that knowledge.

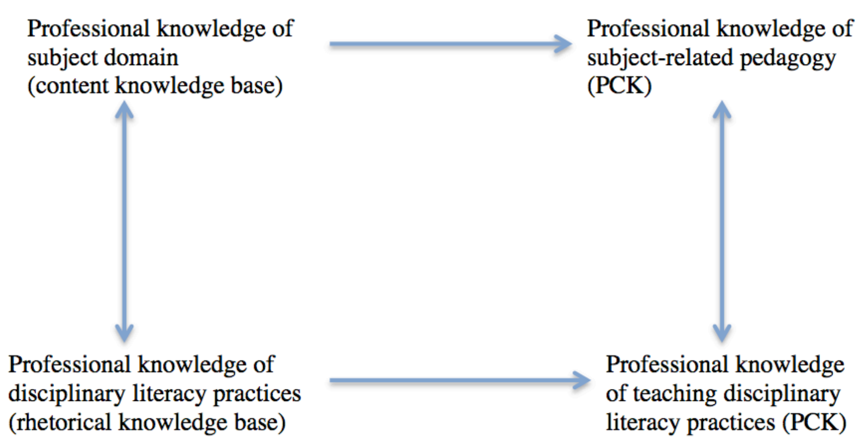

Figure 1. Professional and metalinguistic knowledge

The mastery of mathematics discourse is not straightforward. As Bossé and Faulconer (2008) point out, "Mathematics texts are more conceptually dense that almost any other type of text" (p. 9), given their multimodal nature and the fact that they may call for non-linear reading practices. In addition, they point out, "Writing mathematics often requires a solid understanding or numeric, symbolic, graphical, and verbal representations, their uses, and their interconnections" (p. 9).

Talk and writing in the mathematics classroom

Underpinning the "Culture of Writing" project was a conviction that all teachers need to see themselves as teachers of certain types of writing (Locke & Hawthorne, 2016). In the New Zealand context, however, recent research has indicated that mathematics teachers generally rate themselves as the least efficacious as teachers of writing compared to colleagues in other curriculum areas (Locke & Johnston, 2016). This finding resonates with an assertion by Siebert and Draper (2008), writing in the American context that there is widespread resistance among mathematics teachers "to cooperate in literacy instruction" (p. 229).

Our pilot study was consistent with research, which explores "the effects of particular tools (whether specific words or other forms of representation or more extensive semiotic systems) on the development of mathematical activity" (Morgan et al., 2014, p. 847). While the umbrella project was entitled "A Culture of Writing", it was premised on the desirability of viewing the writing classroom as a community, where talking *about* writing (including writing topics) was an essential practice component. In effect, we subscribed to Douglas Barnes' famous dictum that "Writing floats on a sea of talk" (Simpson, Mercer, & Majors, 2010). In the same issue of *English Teaching: Practice and Critique* where the Simpson et al. editorial occurs, Barnes himself stressed the importance of exploratory talk in encouraging risk-taking and innovation (Barnes, 2010). While the activities Sam developed for his intervention (see below) were focused on writing, they occurred in a context where talk was encouraged and valued.

One research focus relates to the productive use of talk for developing mathematical understanding concerns teacher-pupil interactional patterning. In a 2004 study, Rojas-Drummond and Mercer compared groups of Mexican teachers who achieved good results in mathematics and literacy with teachers who didn't. Their findings indicated that the former group tended to use question-and-answer sequences to help develop student understanding. The latter group was found to practice more traditional questioning techniques. In a systematic review of 15 studies conducted under the auspices of the Evidence for Policy and Practice Information (EPPI) Centre, Kyriacou and Issitt (2008) found that mathematical learning was enhanced when teachers used questioning to probe student reasoning and to invite explanation, and not just to get the right answer.

A second research focus, more pertinent to our study, is on pupil talk in group settings. As Simpson and colleagues (2010) point out, there is a body of literature that calls into question the efficacy of group work for learning. However, they point out, "when students are socialised into the discourses of classroom interactions by teachers—to engage in content specific, reasoned discussions—the quality of group activity and its learning outcomes are greatly improved, to statistically significant levels" (p. 4). In a major study, Mercer and Littleton (2007) found that *effective* [our emphasis] was not only instrumental in improving students' learning outcomes in science and mathematics but also made a statistically significant difference to their reasoning ability.

Examples of practitioner inquiry typically have students explore a range of modes, especially writing, for representing conceptual understanding in mathematics. Based on a range of research findings, for example, Bossé and Faulconer (2008) provide a range of illustrative tasks related to "writing **in** math" (p. 16 [their emphasis]). Also focusing on writing, Vacaretu (2008) describes the development of two instructional strategies where "problem writing enhances students' awareness of the logical structure of mathematics problems and to show how such awareness, in turn, increases success with problem solving" (p. 452). (See also, Baxter, Woodward, Olson, & Robyns, 2002; Fernstein, 2007; Pugalee, 2001; Renne, 2004) Fernstein (2007) advocates a writing workshop approach to developing a community of mathematical practice that in many ways parallels the writing workshop approach in literacy learning and teaching (Lieberman & Wood, 2003). She writes that: "By necessitating social interaction and giving participants an authentic audience, writing workshops provide young mathematicians with a unique learning experience" (Fernstein, 2007, p. 278).

In the New Zealand context, Sharma, Doyle, Shandil and Talakia-atu (2012) prefaced their report on the development of a four-stage framework for assessing statistical literacy by remarking that it was a relatively new arrival in statistics education research. In design research with Year 9 students from two classrooms undertaking the statistics topic, these researchers drew on Gal's (2004) definition of statistical literacy as involving *knowledge elements* (mathematical knowledge, statistical knowledge, context knowledge, literacy skills and critical questions) and *dispositional elements* (beliefs and attitudes, critical stance) (p. 152). In a study building on this research, Sharma (2013) found that "with suitable scaffolding and support students were able to interpret and critically evaluate statistical information" (p. 203) and articulate their understandings to their classmates. A number of strategies—involving both talk and writing—were used to prompt discussion and critical thinking, including pre-prepared key questions, small-group discussions, and reporting back. "Writing support involved writing frames, cloze activities, and composing responses individually and in groups" (Sharma, 2013, p. 204). The following statement from one of the student participants is telling in regard to student dispositions re the place of literacy in mathematics learning: "Because usually, like in normal maths, we don't use literacy … like we use addition, subtraction but we actually have some kind of literacy for the things we do in statistics" (Sharma, 2013, p. 204).

There appears to be little research focused specifically on the place of writing in the teaching of algebra, particularly in the early years of secondary school. An exception is Steele (2005), who studied whether the use of writing would help eight, Grade 7, "pre-algebra" students develop "schemata knowledge [sic] for algebraic thinking", defined as including "the ability to analyse and recognize patterns, to represent the quantitative relationships between the patterns, and to generalize these quantitative relationships" (p. 142). The study was guided by a belief that having students *write* about problem-solving was a way of accessing and utilising schematic knowledge. The research question for this practitioner inquiry was: "In what ways do students write about and use schemata knowledge [sic] when solving algebraic problems related in mathematical structure?" (p. 144). As a result of the experiment, students learned to "communicate their schemata knowledge by explaining and justifying their solutions" (p. 152), developed their conceptual knowledge, and connections between the latter and procedural knowledge. Importantly, Steele writes, "By asking students to write about their thinking and to state their generalizations in their natural language, most were able to then write their generalizations symbolically" (p. 152).

This pilot study drew on the conceptual framework established by such mathematics researchers as Morgan and colleagues (2014) in relation to the place of mathematics discourse in mathematics learning. Such a framework, in relation to subject-related literacies (including writing) also underpinned the writing workshop professional learning that Sam had experienced. We saw ourselves as building on the multimodal work of Sharma and colleagues (2012) in relation to statistics education, but with a focus on algebra. While Steele's (2005) work was of interest to us, our participants were older students for whom algebra was a curriculum topic at Year 9.

The intervention

The context of the pilot was a "manipulation of algebra" unit, which was planned to occupy 4–5 weeks of the 2014 programme. The unit content comprised substitution, simplifying algebraic expressions, forming and solving equations, and expanding and factorising. Students brought a large variation in prior knowledge to this intervention. Most had a little knowledge of algebra, being able to simplify and solve simple equations, but without using formal mathematical writing and with minimal conceptual understanding. A few had no prior knowledge at all. Students in both classes were subject to the intervention. Some consideration was given to having one of the classes function as a control group, but this option was rejected for ethical reasons.[i]

[i] Ethical approval was obtained via a whole-school agreement with the secondary school in question for this project. In addition, name obtained consent from his intervention-class students before any data gathering occurred.

Like other teachers in the umbrella project, Sam worked with a research template that offered a step-by-step guide to practitioner inquiry (see Locke, 2010). The PLC collaboratively formulated specific learning objectives (LOs) for their own respective subjects. The LOs Sam formulated for his case-study students were:

1. Students can explain/express their ideas/thinking about algebra in writing.
2. Students can write algebraically by:
    a. Using algebra to express ideas;
    b. Following the conventions of Mathematical working.
3. Students develop skills in providing peer/group feedback in response to classmates' mathematical writing.

The formulation of such learning objectives (LOs) was the cornerstone for the development of a kind of *teaching and research logic*. We express the logic thus: learning tasks/activities are designed in order to help students achieve the stipulated LOs; data collection methods are determined by the question: "How do we know if this activity actually facilitates the attainment of this or that LO?" While activities were more or less decided on beforehand, in keeping with the dynamic nature of action research, teacher-researchers viewed themselves as open to modifying them in the light of ongoing reflection on how their interventions, as planned, were working out in practice.

Table 1 is an example of the first aspect of this logic and offers an overview of the planning involved and the activities planned.

Table 1. Linking learning objectives with activities and resources

LO	Activity
1	Concept circles "Snowballing" Paired feedback Translating language based on algebraic representation into plain speech (including summary tasks) Translating plain speech into language based on algebraic representation
2	Paired feedback "Snowballing" Translating plain speech into language based on algebraic representation Translating algebraic workings into a conventional layout
3	Paired feedback "Snowballing"

The "snowballing" activity invited students to consider a problem or important aspect of a lesson or a "big idea" by themselves, and then formulate a response in writing. They would then move into pairs to share and discuss what they had written. Sometimes this would involve the pair combining responses and developing in writing a jointly constructed response. The pairs would then combine to make fours, then the fours to make eights. A representative of each group of eight would then write their ideas on the board for the class to discuss and develop a consensus on the best response to the problem, important aspect or "big idea".

Summary tasks were used around five times over the course of the unit. At the end of a lesson students would be asked to write down three things they *had learnt this lesson/found interesting/have to remember* for the next lesson. A similar task was used at the beginning of a topic, or in a revision lesson, where students were asked to write down three things they remembered or thought were most important. These tasks were an important formative assessment strategy.

The concept circle task (see Figure 2 for an example) involved the teacher drawing a circle on the board, dividing it into quadrants and writing a concept in each quadrant. Students were challenged to come up with sentences which brought together two or more of the concepts to show how they saw them relating. The example in Figure 2 occurred in the second lesson of the unit, following a demonstration of this task by Sam**.** This task was done around 15 times during the unit (2–3 times per week).

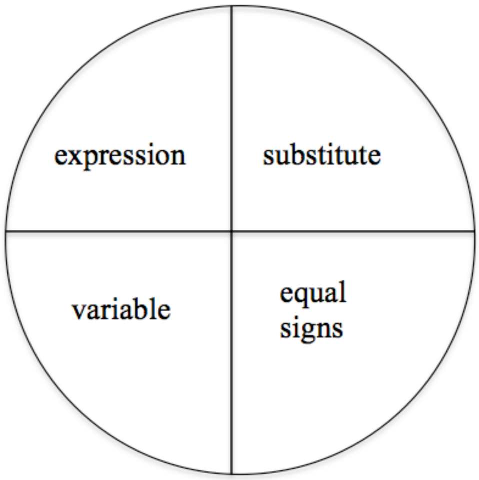

Figure 2. Concept circle

Design and methodology

The overall "Culture of Writing" project[ii] set out to investigate the Writing Workshop-based practices that enhanced teacher self-efficacy in respect of writing and the teaching of writing and, in particular, practices which appeared to have a positive impact on students' (including indigenous Māori students') motivation and performance in writing. It also investigated ways in which a cross-disciplinary "culture of writing" might be seeded and developed in a secondary school. Teachers participating over 2013 and 2014 represented a range of subjects: English, Geography, Science, ESOL, Media Studies, Visual Art and Mathematics. In the context of regular PLC meetings, which included the second author, participating teachers collaborated in drawing on their Writing Workshop experiences to design, introduce and evaluate a writing-focused intervention in just one of their classrooms. In effect, the classrooms these teachers selected became single cases in a collective case study (Yin, 1989; Heigham & Croker, 2009).

In the study reported here, Sam assumed the identity of practitioner researcher, undertaking "systematic, intentional inquiry … about [his] own school and classroom work" by "gathering and recording information, documenting experiences inside and outside of classrooms, and making some kind of written record" (Lytle & Cochran-Smith, 1992, p. 450). As a member of a team undertaking a collective case study, underpinned by a participatory action research ethos (Locke & Hawthorne, 2016), he planned his intervention in consultation with other PLC members.

Action research is not a method *per se* and therefore does not prescribe data collection instrumentation. In this study a range of data collection methods were used in addressing the research questions (see Table 2). As Menter, Elliot, Hulme, Lewin, & Lowden (2011) point out, questionnaires have the advantage of ease of administration, can gather a wide range of information relatively quickly, and allow for the application of a range of statistical procedures. The pre- and post-intervention questionnaires were designed collaboratively by the authors, and aimed to seek student responses to those strategies the second author deemed to be innovative in terms of his usual practice. Items related to confidence were also developed by the authors, and drew on self-efficacy scales widely available online. The 0–5 Likert scale (see Appendix 1) allowed respondents to indicate

[ii] The project was based on a partnership agreement between the school and university that spelled out rights and responsibilities of both parties.

degrees of positivity or negativity towards each item. However, while "ranked categorizing with ordinal data … enables a distinction between higher and lower values", it is limited in that "it can never ascertain the true or exact 'difference' between each response in the scale" (Menter et al., 2011, p. 200). We return to this below.

Table 2. Data collection plan

RQ	Relevant Data
1	Post-intervention questionnaire on response to particular activities used
2	Pre-test and post-test results Plenary worksheets Work samples Teacher reflections and observations
3	Pre-intervention questionnaire on attitude towards writing and algebra (See Appendix 1) Post-intervention questionnaire on attitude towards writing and algebra (See Appendix 1) Teacher reflections and observations

Students in both classes (which we will call 9x and 9y) were given a pre-test on algebra and also a test at the end of the unit. The tests were not identical. The pre-test was focused mostly on identifying prior knowledge of certain algebraic skills, with just a few questions testing understanding or deeper thinking. While the post-test included a number of questions, which tested skills, there was a larger percentage that challenged students to exhibit conceptual understanding.

Analysis and findings

Questionnaires

Algebraic confidence based on Likert ratings

Pre-intervention questionnaire data (see Appendix 2 top table) suggest that prior to the intervention, students were somewhat lacking in confidence in relation to algebra. For instance, 28/49 students graded themselves 3 or above for confidence in writing down mathematical ideas using words and sentences (>3 = 8); and 23/48 graded themselves 3 or above for confidence with algebra (>3=8). In comparison, in the post-intervention questionnaire, 34/44 students graded themselves 3 or above for confidence in writing down mathematical ideas using words and sentences (>3 = 15); and 38/44 graded themselves 3 or above for confidence with algebra (>3=30). The percentage of students who graded themselves 4 or 5 for confidence with algebra increased from 16 percent pre-intervention to 68 percent post-intervention.

As Table 3 indicates, means were calculated for all confidence-related questionnaire items, both pre- and post-intervention. This table provides an analysis of the pre- and post-intervention confidence-related ratings of students in both classes. A number of preliminary comments are in order.

First, this analysis does not constitute a "true experimental" design. For ethical reasons, as mentioned earlier, we decided not to randomly assign students to control and treatment groups. All students received the intervention, so that they were all able to experience the potential benefits of a novel teaching strategy. This precluded the possibility of concluding that the gains obtained were an effect of the intervention and not the influence of uncontrolled variables. However, the design does not prevent the use of the results as evidence that can be triangulated with other evidence such as students' comments (see below). We argue that if numerical findings corroborate the picture generated by other study data, confidence increases in the interpretations we make.

Second, a potential limitation in the data reported in Table 3 is that questionnaires did not identify students' names. We saw anonymity as increasing the likelihood of students providing accurate feedback on their confidence levels. However, this decision prevented our comparing data in the form of "matched pairs", which would have shown each student's individual change in confidence between the pre- and post-tests. This weakens the design in that the two sets of data (pre- and post-ratings) cannot be assessed for their correlation; the presence of correlation tends to increase the likelihood of finding a statistically significant difference between the pre- and post-ratings. Consequently, the "t-test" and "Median Test" findings reported in Table 3 possibly underestimate the change in confidence level of students following the intervention. The absence of matched-pairs comparisons also explains why we selected statistical tests that assumed "independent" data in relation to the pre- and post-test ratings (see Table 3).

A final preliminary comment is that the ratings have been treated as both "ordinal level" and "interval level" data for the purposes of statistical analysis. Ordinal statistical tests (e.g., chi-square, Median Test) are considered to be less powerful (more conservative) than interval statistical tests (e.g., t-test, ANOVA) in that they reduce the likelihood of detecting significant effects when they are present. Strictly speaking, the data should be treated as ordinal (ratings can be arranged from highest to lowest) rather than interval (the distances between any pair of adjacent ratings is the same). The latter implies that moving from a rating of 1 to 2 is the same distance as moving from a rating of 3 to 4; this is a stronger requirement than just simply treating a rating of 2 as greater than a rating of 1. However, Likert scales are often treated as interval in the literature even though, strictly speaking, evidence should be provided that the data meet the requirement of 'equal intervals' between adjacent scale points. The reason for showing both ordinal and interval results here is a recognition of the exploratory nature of this study; it is sufficient for the purposes of triangulation of the ratings with, for example, student comments, to identify the items that have shown more (or less) growth from pre-ratings to post-ratings.

The conclusion we draw from Table 3 is that items 3, 4 and 1 (the latter being more marginal) show greater growth than items 2 and 5. That is, there was greatest growth around confidence with algebra generally (Item 3) and in sharing ideas about algebra with another student (Item 4), some growth in respect of writing down mathematical ideas using words and sentences (Item 1), and marginal in terms of identifiable growth in confidence writing down mathematical ideas using numbers and symbols (Item 2) and feeding back to other students about their ideas on algebra (Item 5). This is more strongly evident from the t-test (interval) results than the Median Test (ordinal) results.[iii] Overall, however, these results support the tone and commentary in the responses to the verbal prompts, which are reported on next.

[iii] Information on the tests referred to in Table 5 is provided in most introductory statistical texts for psychology and education (e.g., Ferguson 1981; Conover 1971).

Table 3. Analysis of mean differences between pre- and post-intervention confidence items

Items	Pretest: (n) Mean Std. Dev.	Posttest: (n) Mean Std.Dev.	Mean Difference (Post – Pre)	t-value* (Signif. level)	Median test with Chi-square value^ (Signif. level)
I am confident to write down my mathematical ideas using words and sentences	(49) 2.57 1.10	(44) 3.07 1.11	0.50	t = 2.18 ($p \leq .05$)	$\chi2 = 3.00$ (Not sig.)
I am confident to write down my mathematical ideas using numbers and symbols	(49) 2.94 1.07	(44) 3.32 1.09	0.38	t = 1.69 (Not sig.)	$\chi2 = 1.61$ (Not sig.)
I feel confident with algebra	(49) 2.20 1.32	(44) 3.50 1.02	1.30	t = 5.25 ($p \leq .001$)	$\chi2 = 23.69$ ($p \leq .001$)
I feel confident sharing my ideas about algebra with another student	(49) 2.16 1.55	(42)# 3.26 0.96	1.10	t = 4.00 ($p \leq .001$)	$\chi2 = 4.35$ ($p \leq .05$)
I feel confident feeding back to another student about their ideas on algebra	(49) 2.61 1.32	(44) 2.95 1.12	0.34	t = 1.35 (Not sig.)	$\chi2 = 0.31$ (Not sig.)

* t-test for independent data; the analysis included application of the F-test for equivalence of variance estimates followed by selection of the appropriate t-model and degrees of freedom (df).

^ The Median Test is also referred to as the Sign Test for Two Independent Samples; it incorporates use of chi-square with df = 1.

Two students did not respond to this item on the post-test.

Algebraic confidence based on questionnaire comments.

As Appendix 1 shows, the pre-intervention questionnaire invited students to complete the sentence starter: "In this topic I would like to learn about …". A thematic analysis of these albeit brief comments indicates a relatively low degree of confidence in algebra. A number of comments express a marked lack of confidence: "How to do algebra cause it's really hard"; "what it is"; "using letters in mathematical terms because that has always confused me"; "algebra in general really I know nothing". Many students responded to this prompt simply by putting the word "algebra". At the other end of the scale, some students showed a preparedness to use mathematics discourse: "the number values relating to letters and how adding something gets you a lower number"; "the x and y variables"; "how to work out what x means when working out sentences".

Post-intervention comments in response to the open-ended prompt, "I enjoyed learning about …" were generally positive (as might be expected given its wording) with the exception of one student who commented: "Nothing, Maths makes me depressed." Most students mentioned specific mathematics topics or terms (such as "algebra" and "variables") or specific operations (e.g. "factorizing", "expanding and substitution", "simplifying" and "solving equations"). Other comments were indicative of a changed attitude to mathematical learning:

- "It was hard at first but now I really enjoy algebra."
- "I enjoyed learning that letters in algebra are terms so they are called variables."

- "Just everything because before even though I had a math tutor last year you helped me way more."
- "I feel confident that I can understand it now."

Overall, then, the post-intervention attitudinal questionnaire responses indicated pronounced positivity towards the learning, and a marked and accurate employment of mathematics discourse compared with the pre-intervention questionnaire (13 students mentioned algebraic concepts in addition to the word "algebra" itself).

Post-intervention questionnaire results: Student responses to selected activities based on questionnaire results.

Post-intervention, students were asked to indicate how helpful they found five of the learning activities that were used during the unit of work. In relation to the scale, 0 indicates, "strongly disagree" while 5 indicates, "strongly agree" along an ordinal scale. Means and medians across both classes for this questionnaire are shown in Table 4.

The post-intervention-only items related to specific activities showed a markedly strong endorsement of working with a partner. Despite the negative comment about textbooks from one student, textbooks were given the next strongest rating for helpfulness from these students. This was followed by a generally positive overall rating of the "little worksheets". On the face of it, the two activities which were conceived as innovative—concept circles and snowballing—received modest or lukewarm endorsement as helping learning. We will discuss this finding later.

Table 4. Post-intervention response to specified activities: Means and medians

Combined Classes (n = 46)	Mean	Median
I found "concept circles" helped my learning a lot.	2.4	2
I found the "snowball activity" helped my learning a lot.	2.6	2.5
I found working with a partner/group helped my learning a lot.	4.7	4
I found working from the textbook helped my learning a lot.	3.5	4
I found the little worksheets at the end of the lesson helped my learning a lot.	3.1	3

There were two open-ended prompts related to specified activities added to the post-intervention questionnaire. The first was: "Other suggestions about what could help my learning…". Of the 24 responses, three called for more time on exercises, five commented on how students were seated and with whom, six called for more explanation and the checking of student understanding, one wanted more practice tests, while two wanted maths to be more fun (e.g. through the use of games).

The second prompt was: "Anything else I would like my teacher to know…". There were 20 responses here (including some silly ones), with 1 also indicating the desirability of better explanation, another calling for more fun, and another reiterating the need for more time on exercises. Most responses (7) indicated that students could not think of anything else they wanted the teacher to do. Additionally, two commented positively on the learning, one indicating that the teacher had done a "good job", and the second commenting: "Your teaching really helped me step up from being behind in Maths, and not be behind the class." Another remarked on what helped his/her motivation: "I feel when you say I have to complete something before we leave I work harder and am more motivated." One student commented that: "Text books are a bit boring."

Test results

Results of both pre-tests and post-tests are shown in Table 5. Clearly, a large majority of students in both classes mastered the unit content, some to a high degree.

Table 5. Pre-test and post-test results

	Not achieved	Achieved	Merit	Excellence
9x pre-test	24			
9y pre-test	21	3		
Pre-test total	45	3		
9x post-test	3	12	8	2
9y post-test	4	13	7	1
Post-test total	7	25	15	3

Teacher reflections and observations

The second author (Sam) did not engage in systematic journaling over the course of his teaching 24 algebra lessons to each class. These reflections, then, are very much his impressions based on lesson-by-lesson notes made on his lesson plans. In summary:

- Because Sam was researching his own practice, he was continually aware of his *own* uses of language, that is, his use of language related to both his content knowledge base and his rhetorical knowledge base (see Figure 1).
- Compared to previous years, in the course of the unit, there were a lot less comments such as, "I don't understand," and "Why are we doing this?"
- Compared to previous years, students seemed more engaged with the ideas and concepts underlying the tasks they were asked to do.
- There appeared to be a much better understanding of algebra, rather than just the ability to use a learnt skill to solve a problem.
- There was a preparedness to use oral language to discuss maths concepts and processes and this was transferred to other topics.
- Both classes had examinations in mathematics in Terms 3 and 4. Compared to previous years, there was far less re-teaching of concepts and operations. There appeared to be far better retention of the topic and hence less need for revision.

Discussion and conclusion

This intervention was premised on the belief that designing activities, which engaged students in using verbal language to talk and write about algebra would help develop and consolidate their mathematics understanding, motivate learning and help retention. In relation to Research Question 1, results would suggest that these activities helped students develop and retain mathematical understanding, particularly on the basis of test scores. Like the participants in Sharma's (2013) study of junior secondary students' development of statistical literacy, these students engaged in *using* talk and written language to develop algebraic understanding and, in particular, to express this understanding to others. With a few exceptions, students performed well in their end-of-topic test and maintained these levels of performance for the remainder of the year.

Following the intervention, and in relation to Research Question 3, students were generally positive about their learning experience and displayed confidence in using algebraic discourse that was not present beforehand. There was statistical corroboration through questionnaire findings of significant changes in students' confidence with algebra, sharing their ideas about algebra with others students, and (to some extent) writing down their mathematical ideas using words and sentences. This confidence was particularly related to self-confidence in using verbal language to express algebraic understanding. Indeed, in numerous post-intervention questionnaire comments, students appeared to relish the opportunity to demonstrate this confidence.

As researchers we certainly found ourselves puzzled about the ways these students rated the five activities we sought their feedback on through the post-intervention questionnaire (Research Question 1). From their perspective, "working with a partner" rated very highly indeed, suggesting that they viewed this engagement as productive in their learning. Such a finding is consistent with Fernstein's (2007) advocacy for a writing workshop approach to developing communities of mathematical practice. It is certainly consistent with our own belief in the importance of talk and collaborative activity in developing mathematical understanding. "Working from the textbook" and the "worksheets" might be viewed as somewhat formal and individual, yet these were rated above the "snowball activity" and "concept circles"; the latter activities certainly would have engaged them in task-focused talk and writing. It may be that these students were predisposed to view "real work" as formal, while less formal, hands-on, talk-focused exercises were not seen as "real work".

As researchers, we concede that there were gaps in the design that can be partly attributed to the intense working environment most classroom teachers inhabit. Growth scores alone do not prove the success of an intervention. However, taken together, the findings reported on above collectively suggest that the intervention was successful, even though we can't be sure of the relative effectiveness of the five particular activities mentioned as items in the post-intervention questionnaire. Future research, already planned, will utilize matched pairs across more classes using mathematics self-efficacy scales as well as test results to give greater validity to findings related to the use of the kinds of activities reported on in this pilot study. A greater use of observations and focus group discussions should also provide a more nuanced picture of how having students engage in *using* language develops their mathematical understanding across a range of topics.

What did Sam take from this pilot study as classroom teacher? In his own self-reflective review, he identified a future need for a stronger literacy component and a need to develop more activities where students share their writing. Looking back on his involvement in the Writing Workshop, he believed that this professional learning had prompted him to teach mathematics writing much more explicitly and to link writing conventions in mathematics with those in English. He viewed the emphasis on writing as prompting him to become aware of other aspects of literacy and (as he wrote in his reflection) this emphasis "energized my teaching of reading and key words/language of algebra". Overall, he believed that his teaching was characterized by a greater "focus on concepts and understandings rather than just skill learning" and that this led to "better understanding and learning of the topic by students".

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Appendix 1: Pre- and Post-intervention questionnaires

Pre-intervention

	Strongly agree					**Strongly disagree**
I am confident to write down my mathematical ideas using sentences	0	1	2	3	4	5
I am confident to write down my mathematical ideas using numbers and symbols	0	1	2	3	4	5
I feel confident with algebra	0	1	2	3	4	5
I feel confident sharing my ideas about algebra with another student	0	1	2	3	4	5
I feel confident feeding back to another student about their ideas on algebra	0	1	2	3	4	5
In this topic I would like to learn about…						

Post-intervention

	Strongly agree					**Strongly disagree**
I am confident to write down my mathematical ideas using sentences	0	1	2	3	4	5
I am confident to write down my mathematical ideas using numbers and symbols	0	1	2	3	4	5
I feel confident with algebra	0	1	2	3	4	5
I feel confident sharing my ideas about algebra with another student	0	1	2	3	4	5
I feel confident feeding back to another student about their ideas on algebra	0	1	2	3	4	5
I found "concept circles" helped my learning a lot.	0	1	2	3	4	5
I found the "snowball activity" helped my learning a lot	0	1	2	3	4	5
I found working with a partner/group helped my learning a lot	0	1	2	3	4	5
I found working from the textbook helped my learning a lot	0	1	2	3	4	5
I found the little worksheets at the end of the lesson helped my learning a lot	0	1	2	3	4	5
I enjoyed learning about…						

Appendix 2: Pre- and Post-intervention questionnaires: Raw data

Pre-intervention questionnaire results: Confidence

Combined Classes (n = 49)	0	1	2	3	4	5
I am confident to write down my mathematical ideas using words and sentences	4	1	16	20	7	1
I am confident to write down my mathematical ideas using numbers and symbols	1	3	12	17	14	2
I feel confident with algebra	6	10	10	15	7	1
I feel confident sharing my idea about algebra with another student	10	6	13	10	6	4
I feel confident feeding back to another student about their ideas on algebra	4	8	5	20	10	2

Post-intervention attitudinal questionnaire results: Confidence

Combined Classes (n = 44)	0	1	2	3	4	5
I am confident to write down my mathematical ideas using words and sentences	2	1	7	19	12	3
I am confident to write down my mathematical ideas using numbers and symbols	2	0	5	16	17	4
I feel confident with algebra	1	2	3	8	28	2
I feel confident sharing my idea about algebra with another student	2	1	4	16	15	2
I feel confident feeding back to another student about their ideas on algebra	3	3	4	20	11	3

Post-intervention results: Response to specified activities

Combined Classes (n = 46)	0	1	2	3	4	5
I found “concept circles” helped my learning a lot	2	4	19	17	3	1
I found the “snowball activity” helped my learning a lot	3	5	15	11	9	3
I found working with a partner/group helped my learning a lot	1	1	5	15	15	9
I found working from the textbook helped my learning a lot	1	0	3	7	26	7
I found the little worksheets at the end of the lesson helped my learning a lot	1	3	6	19	14	3